Early Universe Cosmology the No-Boundary Proposal

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Based on CJ, Lehners [arXiv: 2008.04134]

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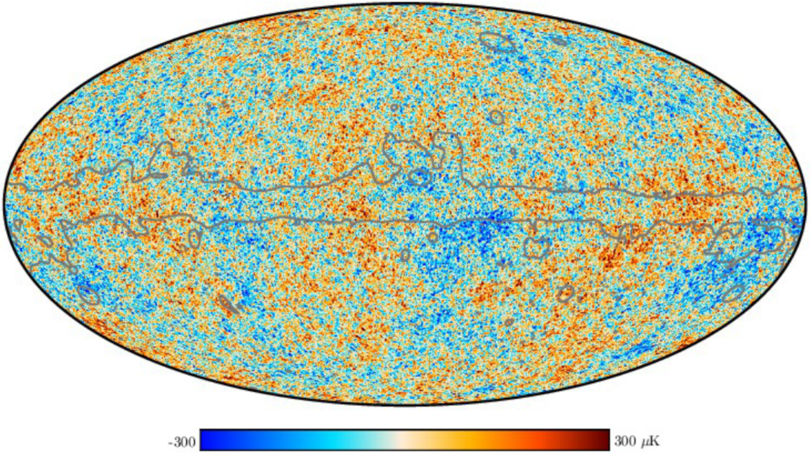
Parallel talk session – Cosmology
Universität Hamburg

The CMB: homogeneity and isotropy of our Universe



2018 SMICA temperature map. Planck Collaboration I 2018

arXiv: 1807.06205.

General Relativity in 3 minutes

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→ Einstein-Hilbert action:

$$S_{\mathsf{EH}} = \frac{1}{16\pi G} \int \mathrm{d}^4x \sqrt{-g} \left(R - 2\Lambda\right)$$

GR applied to cosmology: ΛCDM model

Homogeneous and isotropic spacetime

FLRW metric: $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2$

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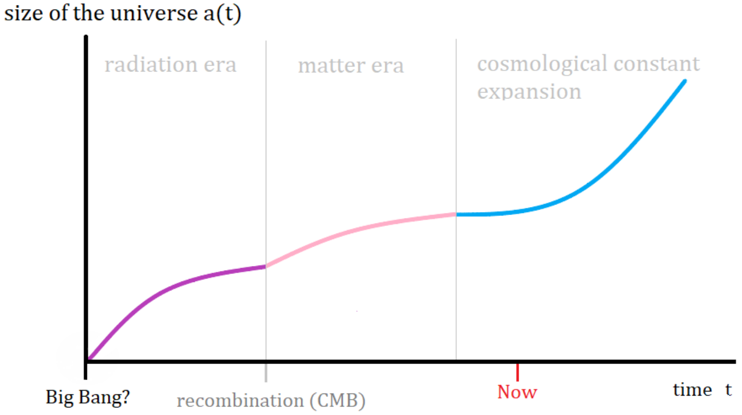

A theory of initial conditions: the no-boundary proposal

Quantum state for the Universe

$$\Psi(\mathsf{now}) = \int_{\mathsf{initial\ state}}^{\mathsf{now}} \mathcal{D}g_{\mu\nu}\mathcal{D}\Phi e^{iS\,[g_{\mu\nu},\Phi]}$$

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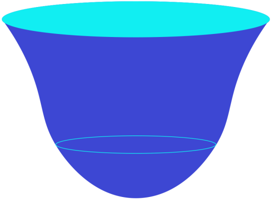
- $a(\tau) = \dfrac{1}{H}\sin(H\tau) \;\rightarrow\; \dot{a} = +i$
- sum over **regular** (Euclidean) and geometries starting at the SP with fixed momentum



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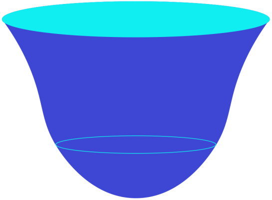

GR cannot be **quantized** → is no-boundary solution **robust** to quantum corrections to GR?

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$\rightarrow$ General type of action we consider:

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \, f\left(R_{\mu\nu\rho\sigma}\right)$$

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  $A_1 = \frac{\dot{a}^2 + N^2}{a^2 N^2}$; $A_2 = \frac{\ddot{a}N - \dot{a}\dot{N}}{a N^3}$

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→ Yes, assuming some symmetry on the Lagrangian:

$$\sum_{p_1, p_2} c_{p_1, p_2} (p_1 - p_2) = 0$$

(quadratic gravity, $f(R)$, low-energy expansion of ST all OK)

The no-boundary solution beyond GR: one example

For type IIB string theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big(R + \alpha'^3\mathcal{E}_{(0,0)}\mathcal{R}^4 + \alpha'^5\mathcal{E}_{(1,0)}\nabla^4\mathcal{R}^4 + \dots\Big)$$

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$$\begin{aligned}
\mathcal{R}^4 =& \, 12(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^2 + 6R^{\mu\nu\rho\sigma}R_{\mu\nu}{}^{\kappa\lambda}(4R_{\kappa\lambda}{}^{\zeta\eta}R_{\rho\sigma\zeta\eta} - R_{\kappa\rho}{}^{\zeta\eta}R_{\lambda\sigma\zeta\eta}) \\
& - 12R_{\mu\nu\kappa\lambda}R_{\rho\sigma\zeta\eta}R^{\mu\nu\rho\kappa}R^{\sigma\lambda\zeta\eta} + \frac{3}{2}R_{\mu\nu\kappa\lambda}R^{\mu\rho\kappa\sigma}R^{\lambda\eta}{}_{\rho\zeta}R^{\nu\zeta}{}_{\sigma\eta} \\
& + \frac{3}{4}R_{\mu\nu\kappa\lambda}R^{\mu\rho\kappa\sigma}R_{\rho\zeta\sigma\eta}R^{\nu\zeta\lambda\eta} \\
=& \, 1467(A_1^2 + A_2^2)^2 + 738(A_1^4 + A_2^4)
\end{aligned}$$

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$\exists$ a no-boundary solution!

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future prospect: general conditions? broader type of actions?

Thank you for your attention!