Contribution ID: **10** Type: **Minisymposium Contribution**

Optimal Control of the Generalized Riemann Problem for Hyperbolic Systems of Conservation Laws

Wednesday, August 14, 2024 10:00 AM (30 minutes)

In this talk, we analyze optimal control problems for quasilinear strictly hyperbolic systems of conservation laws where the control is the initial state of the system. The problem is of interest, for example, in the context of fluid mechanics or traffic flow modelling. Similar problems for scalar conservation laws have already been studied. However, the case of hyperbolic systems is more involved due to the coupling of the characteristic fields.

We begin our analysis by considering the Generalized Riemann Problem, which has a piecewise smooth initial state with exactly one discontinuity. This is a natural choice since it is well known that solutions to hyperbolic conservation laws generally develop discontinuities even for smooth data. For piecewise C^1 initial data we obtain the existence, uniqueness and stability of an entropy solution by a careful fixed point argument built on the associated Riemann Problem with piecewise constant initial states. The construction yields insights into the structure and regularity of the solution and provides a foundation to derive differentiability results of the control-to-state mapping.

The entropy solution is piecewise C^1. Its smooth parts are separated by C^2 curves which are either shock curves or boundaries of rarefaction waves. In a subsequent step, we show that these curves depend differentiably on the initial state. This allows the transformation to a fixed space-time domain as a reference space. In this reference space, we can show that the transformed solution depends differentiably on the initial state in the topology of continuous functions. For this, a detailed knowledge of the structure of the solution and the behaviour of the shock curves is crucial. As an immediate consequence, the differentiability of tracking type functionals for the optimal control problem follows.

Authors: BREITKOPF, Jannik (TU Darmstadt); Prof. ULBRICH, Stefan (TU Darmstadt)

Presenter: BREITKOPF, Jannik (TU Darmstadt)

Session Classification: MS 06: Recent advances in PDE-constrained optimization

Track Classification: Minisymposia: MS 06: Recent advances in PDE-constrained optimization