

A relaxation-based probabilistic approach for PDE-constrained optimization under uncertainty with pointwise state constraints

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We consider a class of convex risk-neutral PDE-constrained optimization problems subject to pointwise control and state constraints. Due to the many challenges associated with almost sure constraints on pointwise evaluations of the state, we suggest a relaxation via a smooth functional bound with similar properties to well-known probability constraints. First, we introduce and analyze the relaxed problem, discuss its asymptotic properties, and derive formulae for the gradient the adjoint calculus. We then build on the theoretical results by extending a recently published online convex optimization algorithm (OSA) to the infinite-dimensional setting. Similar to the regret-based analysis of time-varying stochastic optimization problems, we enhance the method further by allowing for periodic restarts at pre-defined epochs. Not only does this allow for larger step sizes, it also proves to be an essential factor in obtaining high-quality solutions in practice. The behavior of the algorithm is demonstrated in a numerical example involving a linear advection–diffusion equation with random inputs. In order to judge the quality of the solution, the results are compared to those arising from a sample average approximation (SAA). This is done first by comparing the resulting cumulative distributions of the objectives at the optimal solution as a function of step numbers and epoch lengths. In addition, we conduct statistical tests to further analyze the behavior of the online algorithm and the quality of its solutions.

Authors: KOURI, Drew; STAUDIGL, Mathias; SUROWIEC, Thomas

Presenter: STAUDIGL, Mathias

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