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Constrained Best Approximation of Shape Tensors and its Role for the Determination of Shape Gradients

Thursday, August 15, 2024 9:00 AM (30 minutes)

A crucial issue in numerically solving PDE-constrained shape optimization problems is avoiding mesh degeneracy. Recently, there were two suggested approaches to tackle this problem: (i) departing from the Hilbert space towards the Lipschitz topology approximated by W^{1,p^*} with $p^* > 2$ and (ii) using the symmetric rather than the full gradient to define a norm.
In this talk we will discuss an approach that allows to combine both. It is based on our earlier work [2] on the L^p approximation of the shape tensor of Laurain & Sturm [1]. There we have shown that W^{1,p^*} shape gradients can be determined as Lagrange multipliers of this L^p approximation problem. We extend this by adding a symmetry constraint to the derived L^p least mean approximation problem and show that the distance measured in a suitably weighted L^p-norm is equal to the dual norm of the shape derivative with respect to the L^{p^*}-norm associated with the linear elastic strain of the deformation. The resulting L^p least mean problem can be viewed as a generalization of a constrained first-order least squares formulation. In addition, as in the case without symmetry constraint, it turns out that the Lagrange multiplier associated with the divergence constraint is the direction of the steepest descent, but now with respect to the norm defined by the symmetric gradient. This provides a way to compute shape gradients in W^{1,p^*} with respect to this elasticity type norm. The discretization of the resulting least mean problem can be done by the PEERS element and its three-dimensional counterpart. We will illustrate the advantages of this approach by computational results of some common shape optimization problems.

References:

[1] A. Laurain and K. Sturm. Distributed shape derivative *via* averaged adjoint method and applications. *ESAIM Math. Model. Numer. Anal.*, 50(4):1241–1267, 2016.

[2] G. Starke. Shape optimization by constrained first-order system mean approximation, 2023. arXiv:2309.13595.

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