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Bilinear control problems governed by PDEs: analysis, optimization and discretization

Thursday, August 15, 2024 2:00 PM (1 hour)

Despite its importance in applications, there are not many references dealing with the analysis of bilinear control problems governed by elliptic partial differential equations.

In this talk, we will deal with an optimal control problem where the control acts in a multiplicative way. We will investigate both the case in which the control is the reaction coefficient (distributed) and the case in which the control acts as the Robin coefficient on the boundary.

First we will perform a careful study of the control-to-state mapping. Differentiability in L^s for some appropriate s>=2 is not immediate. We are able to prove it and this allows us to obtain first order necessary optimality conditions and no-gap second order sufficient optimality conditions for the control problem.

Next we rewrite the problem using a semismooth equation and prove superlinear convergence of the semismooth Newton method in an infinite-dimensional framework. This is done under the assumptions of no-gap second order sufficient optimality conditions and strict non-complementarity, as is usual for this kind of algorithms in the finite-dimensional setting. Previous results in the literature assumed local convexity, which is a rather stringent assumption for bilinear control problems. An algorithm is provided. The 3D boundary problem represents a challenge due to the lack of differentiability in L^2.

Finally, we discretize the problem using finite elements. We will show results for the distributed case. Piecewise constant functions are used to approximate the controls, while continuous piecewise linear functions are chosen to approximate both the state and the adjoint state variables. Convergence and error estimates are obtained assuming no-gap second order sufficient optimality conditions. For the control, the obtained order of convergence is O(h), while for the state and adjoint state we are able to prove order O(h^2). This kind of behaviour where the order of convergence found in numerical experiments for the state is better than the one expected from the error in the control is very usual in control problems. It is normally referred to as superconvergence.

Numerical experiments that confirm all our findings will be presented.

This conference gathers results obtained in collaboration with Eduardo Casas and Konstantinos Chrysafinos.

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