CRC1624 Summer School
”Algebra and Geometry meets
Quantum Fields and Strings”

Collection of Abstracts

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Contents

1. Lectures

1.1 Lara Anderson: Geometric Transitions and string compactifications

This lecture series will focus on topology changing transitions in Calabi-Yau and related geometries and the realization of such transitions in effective field theories arising from string theory. This will include classic examples in the context of 4- dimensional, N=2 compactifications (including conifold and flop transitions in Type II theories) and more recent examples in 4-dimensional, N=1 theories (including heterotic string theory and F-theory).

1.2 Sabin Cautis: Categorical structure of Coulomb branches

Coulomb branches are certain moduli spaces arising in supersymmetric field theory. They include as special cases many spaces of independent interest in geometry and representation theory. In the four-dimensional case, their coordinate rings can be categorified (lifted) to tensor categories that carry a lot of structure. In these talks we will define and explore the structure of such categorified Coulomb branches.

1.3 Dustin Clausen: Real-analytic derived categories and applications

I will describe a certain derived category, extracted from joint work with Peter Scholze, which serves as a reasonable "base category" for real-analytic geometry. Then I will illustrate its use in several examples.

Lecture 1: Real-analytic base categories Lecture 2: Trace-class maps, nuclear objects, and rigid categories Lecture 3: Quasi-coherent and elliptic sheaves Lecture 4: Analytic K-theory Lecture 5: Quantum field theories, after Kontsevich-Segal

1.4 Tristan Collins: Stablility and PDEs in mirror symmetry

This lecture series will introduce the special Lagrangian and deformed Hermitian- Yang Mills equation, nonlinear geometric PDEs arising in the context of mirror symmetry. The expectation as that the solvability of these equations is related to algebro-geometric notions of stability. We will explain some historical developments motivating this picture and discuss some more recent progress.

1.5 Clay Cordova: Introduction to Non-Invertible Symmetry

We discuss foundational examples of non-invertible symmetry in quantum field theory. These include examples in 2d conformal field theories and constraints on renormalization group flows, as well as examples in 4d gauge theories involving chiral symmetry and electric-magnetic duality.

1.6 Michele Del Zotto: Generalized Symmetries and TQFTs

In this course I will introduce generalized symmetries of quantum fields and explore some of their consequences on dynamics. The course is divided in two parts. The first part focuses on so-called p-form symmetries. I will introduce their basic properties, discuss spontaneous symmetry breaking, and explain some applications to confinement in gauge theories. If time permits, I will also introduce higher-group symmetries, symmetry fractionalization and symmetry transmutation. The second part develops a complementary perspective. I will introduce the topological symmetry theory, a topological quantum field theory (TQFT) in one higher dimension that one can use to characterize generalized symmetries, their anomalies, and their action on extended operators and defects. Starting from the familiar idea of anomaly inflow, I will give a gentle introduction to this framework and illustrate it with applications to symmetry-preserving gapped phases and to the construction of duality defects for (3+1)-dimensional theories. The course is intended to provide an introduction to the rapidly developing language of generalized symmetries, presenting materials complementary to the ones discussed in the lectures by Clay Córdova at this school.

1.7 Pavel Etingof: Analytic Langlands correspondence

Let G is a connected reductive group and X a smooth irreducible projective curve, both defined over a field F. The goal of *the global Langlands program over function fields* is to do harmonic analysis on the moduli space (stack) $\mathrm{Bun}_G(X)$ of principal G-bundles on X, for various fields F. This means that we want to find the spectral decomposition for a family of commuting Hecke operators (or functors) acting on a suitable space of functions (or category of sheaves) on $\mathrm{Bun}_G(X)$. More precisely, the Langlands program aims to parametrize this spectrum by data related to the *Langlands dual group* $G^\vee$, namely appropriate $G^\vee$-*local systems* on X. The corresponding eigenvectors (or eigensheaves) are called *automorphic functions (sheaves)*. There is also a ramified generalization, in which one considers G-bundles with level structure at a finite set of points of X.

In the most classical setup, developed by Langlands himself around 1970 (the *arithmetic Langlands program*), F is a finite field $\mathbb{F}_q$, so $\mathrm{Bun}_G(X)(F)$ is an infinite countable set, and one considers Hecke operators $H_{x,\lambda}$ acting on $L^2(\mathrm{Bun}_G(X)(\mathbb{F}_q))$ (a family of bounded, commuting normal operators). In this case, the Langlands conjecture (a version of which was recently proved by D. Gaitsgory and S. Raskin) states that the spectrum of Hecke operators is parametrized by local systems on X with values in $G^\vee$ (or representations of the absolute Galois group of the field $\mathbb{F}_q(X)$), understood in a suitable sense.

In 1990s Beilinson and Drinfeld proposed a geometric generalization of this theory (the *geometric Langlands program*). In this generalization, $F = \mathbb{C}$, and the role of the space $L^2(\mathrm{Bun}_G(X)(\mathbb{F}_q))$ is played by the category of *(critically twisted) D-modules* (de Rham version) or *constructible sheaves* of $\mathbb{C}$-vector spaces (Betti version) on the stack $\mathrm{Bun}_G(X)$; the two versions are related by the Riemann-Hilbert correspondence. On this category there is an action of a family of commuting *Hecke functors*, and the goal of the geometric Langlands program is to "diagonalize" these functors by finding and parametrizing their eigensheaves. The geometric Langlands conjecture of Beilinson and Drinfeld was that such eigensheaves are parametrized by $G^\vee$-local systems on the Riemann surface $X(\mathbb{C})$, i.e., in the de Rham version, holomorphic $G^\vee$-bundles on X with a connection, and in the Betti version, finite dimensional complex representations of its fundamental group. This conjecture has recently been proved in general by D. Gaitsgory and his collaborators. Also, Kapustin and Witten showed that the geometric Langlands program is related to topologically twisted 4-dimensional $N = 4$ supersymmetric gauge theory.

There is also a version of this theory in which F is any field and the constructible sheaves are ℓ-adic (i.e., of $\overline{\mathbb{Q}}_\ell$-vector spaces). This version, with $F = \mathbb{F}_q$ where $(\ell, q) = 1$, connects to the arithmetic version via the *sheaf-function correspondence* (i.e., taking the trace of Frobenius in the étale cohomology of the sheaf to get a function). This gives ℓ-adic, rather than complex, functions, but it does not matter too much since cuspidal automorphic functions (i.e., ones from the discrete spectrum of Hecke operators) take values in $\overline{\mathbb{Q}}$.

In the de Rham version, the automorphic D-module M_L attached to a $G^\vee$-local system L is easiest to describe when L is an *oper*, i.e., a connection on a special *oper* $G^\vee$-bundle E, induced from the oper bundle for the principal SL_2 of $G^\vee$, which is (for genus$(X) \neq 1$) the non-trivial extension of $K_X^{-1/2}$ by $K_X^{1/2}$. The space of opers, $\mathrm{Op}_{G^\vee}(X)$, is an affine space whose underlying vector space is the *Hitchin base* $\mathcal{B} = \oplus_{i=1}^r H^0(X, K_X^{d_i})$, where d_i are the degrees of basic invariants for G and r is its rank. Beilinson and Drinfeld showed that the classical Hitchin system $\pi : T^*\mathrm{Bun}_G(X) \to \mathcal{B}$ admits a quantization, which gives an inclusion (in fact, an isomorphism) $D : \mathbb{C}[\mathrm{Op}_{G^\vee}(X)] \to \mathcal{D}(\mathrm{Bun}_G(X), K^{1/2})$ (differential operators on half-forms, i.e., critically twisted ones). This quantization is constructed using the Casimirs of the affine Lie algebra at the critical level, constructed around 1990 by B. Feigin and E. Frenkel. The twisted D-module M_L is then defined by the system of *quantum Hitchin differential equations*

$$D(f)\psi = f(L)\psi,\ f \in \mathbb{C}[\mathrm{Op}_{G^\vee}(X)].$$

Finally, in the last 7 years, jointly with E. Frenkel and D. Kazhdan, we developed an *analytic version* of the Langlands program. This program develops the earlier ideas and results of Braverman-Kazhdan, Kontsevich, Langlands and Teschner. In this version, F is a local field, so there are two cases: archimedean ($F = \mathbb{R}, \mathbb{C}$) and non-archimedean ($F$ is a finite extension of $\mathbb{Q}_p$ or $F = \mathbb{F}_q(\!(t)\!)$). In both cases, we are interested in the spectrum of Hecke operators $H_{x,\lambda}$ acting on the Hilbert space $L^2(\mathrm{Bun}_G(X)(F))$, where $\mathrm{Bun}_G(X)(F)$ is now generically an analytic F-variety, and L^2 stands for the space of square integrable half-densities. The operators $H_{x,\lambda}$ are densely defined on the Hilbert space, and conjecturally extend to bounded (in fact, compact) commuting normal operators; this conjecture has been proved in the ramified setting for genus 0 and 1 for $G = PGL_2$.

This raises the question what parametrizes the spectrum of the Hecke operators: can we describe it in terms of the Langlands dual group? In the non-archimedian case, we don't even have a conjecture how to do this, even though it has been shown by Braverman, Kazhdan and Polishchuk that cuspidal automorphic functions of arithmetic Langlands (i.e., over $\mathbb{F}_q$) can be lifted to ones over a local field F with residue field $\mathbb{F}_q$ if we fix a lift of X over its ring of integers $\mathcal{O}_F$.

The situation is better in the archimedian case, however. In this case, a prominent role is played by quantum Hitchin operators, which (conjecturally) act on $L^2(\mathrm{Bun}_G(X)(F))$ as commuting unbounded normal operators and commute with Hecke operators, hence have the same spectral decomposition. This connects the story to geometric Langlands, showing that automorphic functions of analytic Langlands theory should be parametrized by a certain (discrete) family of $G^\vee$-opers. And indeed, for $F = \mathbb{C}$, these (conjecturally) turn out to be the *opers with real monodromy*, i.e., ones whose monodromy representation can be conjugated into the split real form $G^\vee(\mathbb{R})$. For such opers, the automorphic function ψ on $\mathrm{Bun}_G(X)(\mathbb{C})$ is simply a single valued analytic joint eigenfunction of $D(f)$ and $\overline{D(f)}$, which is unique up to scaling:

$$D(f)\psi_L = f(L)\psi_L,\ \overline{D(f)}\psi_L = \overline{f(L)}\psi_L.$$

Gaiotto and Witten showed that this conjecture is compatible to the structure of branes in the Kapustin-Witten theory. Also, this conjecture is proved in genus zero and (almost) in genus 1 in the ramified case, for $G = PGL_2$; the last steps in genus zero were made in a recent paper of Ambrosino and Teschner using Sklyanin's separation of variables transform. If $F = \mathbb{R}$, the situation is more complicated (as it depends on the structure of real ovals on X, real forms of G, etc.) but still in many cases one can formulate the condition on the oper (always a topological one, i.e., in the Betti realization) which makes it a point of the spectrum.

In the simplest case of PGL_2 and genus zero with tame ramification at 4 points, $\mathrm{Bun}_G(X)$ is 1-dimensional and the eigenfunctions ψ_L have the form

$$\psi_L(z, \overline{z}) = \eta_1(z)\overline{\eta_2(z)} + \eta_2(z)\overline{\eta_1(z)},$$

where η_1, η_2 is a basis of solutions of a Heun equation (in the most basic case, Lamé equation).

In the mini-course I will review this theory and consider various examples of arising spectral decompositions.

References

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2. P. Etingof, E. Frenkel, and D. Kazhdan, Hecke operators and analytic Langlands cor- respondence for curves over local fields, Duke Math. Journal 172 (2023) 2015-2071, arXiv:2103.01509.

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4. P. Etingof, E. Frenkel, and D. Kazhdan, A general framework for the analytic Langlands correspondence, arXiv:2311.03743.

5. P. Etingof, H. Liu, Hitchin systems and their quantization, arXiv:2409.09505.

1.8 Laura Fredrickson: Hyperkähler Metrics from Riemann–Hilbert Problems

Yau's solution of the Calabi conjecture tells us that Ricci-flat metrics exist in abundance, but its proof tells us nothing about what those metrics look like. Gaiotto, Moore and Neitzke proposed a remarkable answer to the question "what do they look like?" for hyper-Kähler manifolds fibered by complex tori, in the limit in which the fibers collapse. Their prescription is a Riemann–Hilbert problem: one seeks a Undefined control sequence $\mathbb{P}^1$-family of holomorphic Darboux coordinates with prescribed jumps along rays determined by the central charges and enumerative invariants satisfying the Kontsevich–Soibelman wall-crossing formula. It is the hyperähler upgrade of Strominger–Yau–Zaslow: not merely a complex structure on the mirror, but the metric itself is computed from integers. These five lectures give a self-contained account of a program to make the Gaiotto–Moore–Neitzke's prescription rigorous. The first lecture introduces and motivates Gaiotto–Moore–Neitzke's conjecture. The second lecture introduces hyperkähler geometry. The third lecture sets up the Gaiotto–Moore–Neitzke framework, working the Ooguri–Vafa geometry out completely. The fourth lecture carries the program out in full for the "pentagon", the first non-trivial example. The fifth lecture is in two halves: first, what goes wrong for a general BPS spectrum and how we propose to handle it; second, I'll discuss Gaiotto–Moore–Neitzke's conjecture that the metric produced is Hitchin's metric on the moduli space of Higgs bundles.

1.9 Marco Gualtieri: Higher groupoids and generalized Kähler geometry

The traditional description of the 2-dimensional sigma model with N=(2,2) supersymmetry involves a target geometry known as a generalized Kähler metric. Recently, we have understood that this description is only infinitesimal in nature, and that a full description of the model requires the specification of analogues of principal bundles in the form of holomorphic symplectic 2-groupoid principal bundles. We shall see how such higher groupoids may be constructed using quasi-Hamiltonian reduction techniques in the case of the WZW model.

1.10 Melissa Liu: Gromov-Witten theory of Calabi-Yau threefolds

The lecture series will begin with an introduction to Gromov-Witten theory, which can be viewed as a mathematical theory of A-model topological string on Kähler manifolds. The discussion will then specialize to Calabi-Yau threefolds and survey recent progress on higher genus Gromov-Witten invariants of compact Calabi-Yau threefolds. The lecture series will next describe mathematical theories and generalizations of two algorithms for computing all-genus open and closed Gromov-Witten invariants of toric Calabi-Yau threefolds (which are non-compact): (1) the Topological Vertex (proposed by Aganagic-Klemm-Mariño-Vafa) based on the large-N duality relating topological string theory to Chern-Simons theory, and (2) the Remodeling Conjecture (proposed by Bouchard-Klemm-Mariño-Pasquetti) based on mirror symmetry and the Chekhov- Eynard-Orantin

Topological Recursion. The lecture series will conclude with structural results on higher genus Gromov- Witten invariants for toric Calabi-Yau threefolds, including the all-genus Crepant Transformation Conjecture, and Bershadsky-Cecotti-Ooguri-Vafa (BCOV) holomorphic anomaly equations.

1.11 Boris Pioline: Methods for BPS state counting in type II strings on Calabi-Yau threefolds

Four-dimensional string vacua and quantum field theories with supersymmetry exhibit a rich spectrum of supersymmetric bound states, which depends sensitively on the values of the moduli fields or gauge theory parameters. In the context of type II string theories compactified on a Calabi-Yau threefold, the index counting these states coincides with the Donaldon-Thomas invariants associated to the derived category of coherent sheaves in type IIA, or the Fukaya category in type IIB. The wall-crossing formula in principle determines the indices in any chamber, provided they are known in a particular chamber. More generally, the attractor flow tree formula (or equivalently, the scattering diagram in the space of stability conditions) determines them in terms of the so-called attractor indices. For toric CY3 singularities, the category of BPS states is isomorphic to the derived category of quiver representations, and the attractor indices can be determined exactly, giving a complete characterization of the spectrum of BPS states in any chamber. For compact CY3 manifolds such as the quintic, the attractor invariants cannot be determined in general, but certain vanishing results allow to determine the so-called rank 0 Donaldson-Thomas invariants counting D4-D2-D0 brane bound states, in terms of the rank 1 DT invariants, in turn related to Gopakumar-Vafa invariants. These methods in particular provide precision tests of the (mock) modularity properties of generating series of D4-D2-D0 indices (or, in the local case, Vafa-Witten invariants) predicted by S-duality.

1.12 Nick Rozenblyum: Around the geometric Langlands correspondence

The geometric Langlands program connects representation theory, algebraic geometry, and ideas from quantum field theory. In these lectures, I will describe some of the relevant spaces and structures responsible for these connections. A central theme will be factorization in various incarnations - the principle that data attached to several points interact locally when the points collide and become independent when they separate. In physics, this encodes the principle of locality in QFT. In geometric Langlands, factorization provides a powerful local-to-global principle that plays a central role in the theory.

1.13 Ben Webster: 3-d mirror symmetry

I'll give a high level survey of what's known about 3d mirror symmetry from a mathematical perspective and the remaining challenges in the area.

2. Colloquium talks

2.1 Tomoyuki Arakawa: 4D/3D QFT and representation theory

4D/3D quantum field theory in theoretical physics is conceptually rich and gives rise to many interesting mathematical structures, even though a fully rigorous mathematical formulation of the theories themselves is still lacking. A relatively recent discovery by Beem et al. shows that to every 4D N=2 superconformal field theory one can associate a representation-theoretic object called a vertex algebra, which serves as an invariant (or observable) of the theory. Although vertex algebras are inherently algebraic, those arising as invariants of 4D QFT display striking connections with certain geometric objects that also appear as invariants of the same physical theories. Similarly, to each 3D N=4 gauge theory one can associate two vertex algebras, that is, the A-twisted and B-twisted boundary VOAs, by the work of Gaiotto and his collaborators, which may be viewed as refinements of the Higgs and Coulomb branches. In this talk, I will discuss some representation-theoretic aspects of these phenomena.

2.2 Pierrick Bousseau: Holomorphic Floer theory and Donaldson–Thomas invariants.

Holomorphic Floer theory is a complex analogue of Floer theory, formulated for pairs of holomorphic Lagrangian submanifolds in holomorphic symplectic manifolds. I will explain a conjectural relation between holomorphic Floer theory and Donaldson–Thomas invariants. From a physical perspective, the BPS states of four-dimensional N=2 quantum field theories are expected to be recovered from the holomorphic Floer theory of the associated Seiberg–Witten integrable system. For theories arising from non-compact Calabi–Yau threefolds, where counts of BPS states are captured by Donaldson–Thomas invariants, this predicts a description of these invariants in terms of counts of holomorphic curves in complex integrable systems. I will also discuss how several established correspondences between Donaldson–Thomas invariants and logarithmic Gromov–Witten invariants provide evidence for this picture.

2.3 Ron Donagi: Some Classification Questions: Algebraic Geometry vs. SQFT

Seiberg and Witten showed that some supersymmetric quantum field theories can be solved mathematically. Subsequent work suggested that every N=2 susy theory in 4D leads to an algebraically completely integrable system, which can be solved in terms of theta functions. The base of this system is the Coulomb branch of the theory, a vector space whose dimension is called the rank. The theory has a symmetry governed by the 'flavor' Lie algebra. A heroic series of works of Argyres et al gave a complete classification of the resulting integrable systems in rank 1. (Depending on exactly what you count, they get 35 or 28 or 27 such systems.)

In equally heroic works on the algebraic geometry side, Persson and Miranda classified all 279 types of rational elliptic surfaces in terms of their configurations of singular fibers. Subsequent work of Oguiso-Shioda determined the corresponding Mordell-Weil groups of sections, and Karayayla's thesis determined the automorphism groups.

In this talk we will explore the math/physics dictionary. The integrable systems arising from SQFT live on rational elliptic surfaces. Imposing some very simple constraints reduces the Persson-Miranda list to the physicists' list. Some natural expansions of this list arise from Karayayla's work. The flavor symmetry is interpreted in terms of the Mordell-Weil group. Many questions remain open.

2.4 John Francis: Factorization homology and Chern–Simons theory

The theory of factorization homology, developed in joint works with David Ayala, is a higher-categorical form of integration. It integrates quantum-algebraic structures (such as E_n algebras or braided monoidal categories) over manifolds in a way analogous to, and sometimes capturing, functional integration. I'll describe joint work with Kevin Costello and Owen Gwilliam on the interaction of factorization homology with the Chern–Simons theory of 3-manifolds. In this case, factorization homology of a particular E_3-algebra (quantizing the trivial flat connection) gives knot invariants, including the Jones polynomial. Work in progress with Owen Gwilliam and Rajiv Kaipa globalizes this theory, away from the trivial flat connection, to further quantize the stack $\mathrm{LocSys}_G(M)$ of G-local systems on 3-manifold.