Design of a Runge-Kutta based time-stepping algorithm for the NEMO ocean model

Nicolas Ducousso

Florian Lemarié, Gurvan Madec, Laurent Debreu

Context

NEMO solves the non-linear, hydrostatic, primitive equations

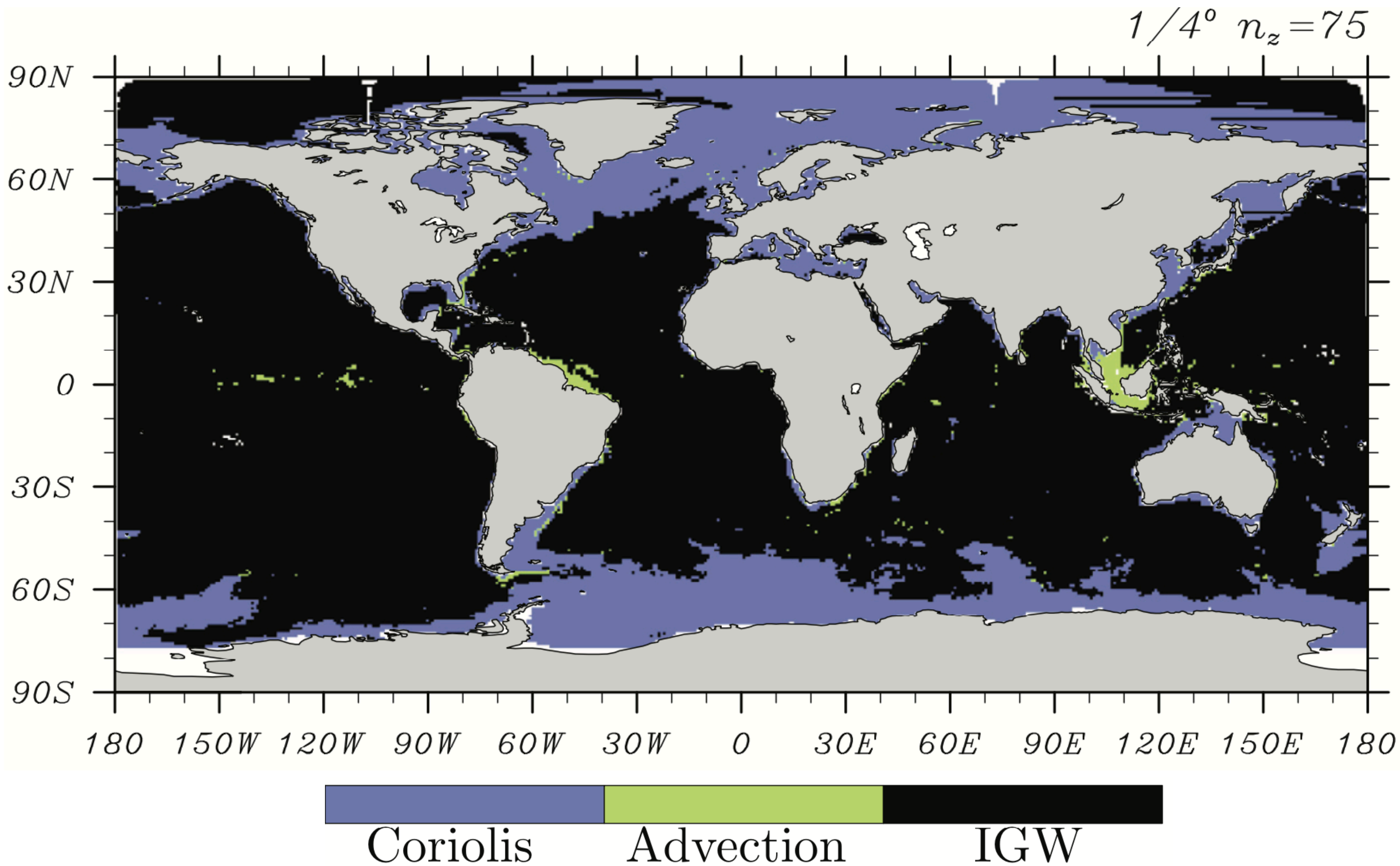
since the 80's :

- finite-difference approach
- C-grid staggering on the horizontal, Lorenz-grid staggering on the vertical
- Leapfrog time stepping + Robert-Asselin time filtering -> LFRA scheme

more recently :

- non-linear free-surface, with z* (and z tilde) vertical coordinates
- fast free surface dynamics treated with a mode-splitting algorithm
- high-order (possibly non linear and adaptive implicit) advection schemes

Context



Lemarié et al., OM 2015

Objectives

revise the time-stepping algorithm to improve the model stability/accuracy/efficiency

LFRA alternatives : AB3 (Durran), LF-AM3 Pred-Cor (Shchepetkin McWilliams), one step, forward-in-time, Runge-Kutta family (Hundsdorfer, Wicker Skamarock)…

RK3 scheme (Wicker Skamarock, 2002) has been favored in several NWP models :

- accurate,
- low storage,
- good stability properties for centered and upwind-biased advection,
- allow a stable time-splitting procedure.

Objectives

revise the time-stepping algorithm to improve the model stability/accuracy/efficiency

LFRA alternatives : AB3 (Durran), LF-AM3 Pred-Cor (Shchepetkin McWilliams), one step, forward-in-time, Runge-Kutta family (Hundsdorfer, Wicker Skamarock)…

RK3 scheme (Wicker Skamarock, 2002) has been favored in several NWP models :

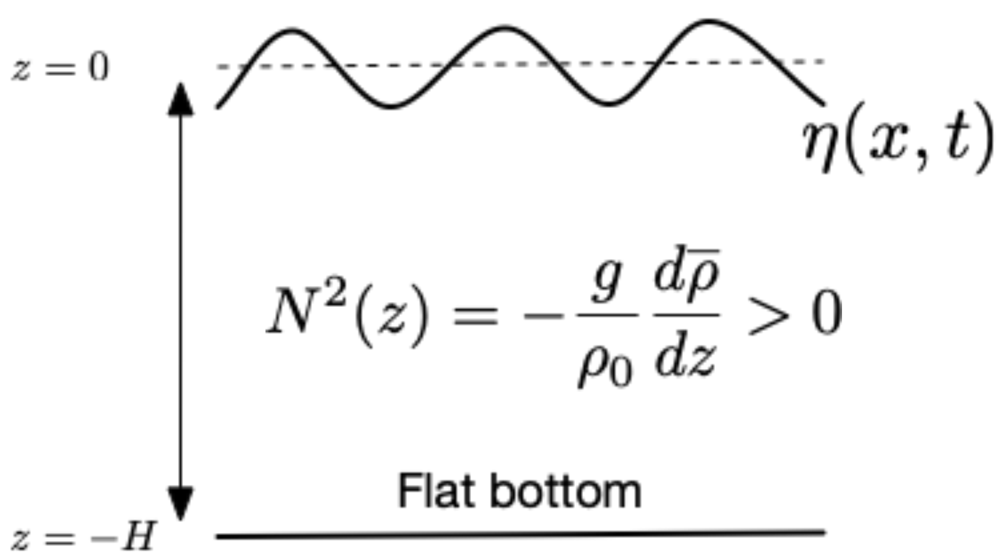
- accurate,
- low storage,
- good stability properties for centered and upwind-biased advection,
- allow a stable time-splitting procedure.

in the following, I investigate the relevancy of RK3 for use in an OGCM

- the analysis framework : normal mode decomposition
- von Neumann linear stability analysis of time-stepping schemes
  - Shallow Water equations
  - Hydrostatic Primitive equations with mode-splitting algorithm
- an idealized numerical experiment

1 - The analysis framework

Starting from the inviscid, adiabatic, hydrostatic primitive equations, in a 2d (x,z) plane, linearized about a resting state, and in absence of rotation :



$$
\left\{
\begin{array}{rcl}
\partial_t u + \frac{1}{\rho_0}\partial_x p & = & 0 \\
\partial_z p & = & -\rho g \\
\partial_x u + \partial_z w & = & 0 \\
\partial_t \rho + w\frac{d\overline{\rho}}{dz} & = & 0
\end{array}
\right.
$$

$$
\begin{array}{rcl}
w(z=0) & = & \partial_t \eta \\
p(z=0) & = & \rho_0 g \eta \\
w(z=-H) & = & 0
\end{array}
$$

1 - The analysis framework

The solution can be decomposed using the vertical normal modes

(Gill 1982, Demange et al. 2019)

$$
\begin{cases}
u(x,z,t) &=& \displaystyle\sum_{q=0}^{\infty} u_q(x,t) M_q(z) \\
p(x,z,t) &=& \displaystyle g\rho_0 \sum_{q=0}^{\infty} h_q(x,t) M_q(z)
\end{cases}
$$

$$
\begin{cases}
\rho(x,z,t) &=& \displaystyle -\rho_0 \sum_{q=0}^{\infty} h_q(x,t) \frac{dM_q}{dz}(z) \\
w(x,z,t) &=& \displaystyle -\sum_{q=0}^{\infty} \partial_x u_q(x,t) \int_{-H}^{z} M_q(s) ds \\
\eta(x,t) &=& \displaystyle \sum_{q=0}^{\infty} h_q(x,t) M_q(0)
\end{cases}
$$

1 - The analysis framework

The vertical modes are the eigenfunctions of the Sturm-Liouville problem :

$$\begin{cases} \Lambda M_q(z) & = & c_q^{-2} M_q(z) \quad \text{with} \quad \Lambda = -\dfrac{d}{dz}\left(N^{-2}\dfrac{d}{dz}\right) \\ \dfrac{dM_q}{dz}(z=0) & = & -\dfrac{N^2(0)}{g} M_q(0) \\ \dfrac{dM_q}{dz}(z=-H) & = & 0 \end{cases}$$

All vertical modes are depth dependent, including the barotropic/external mode :

with $N^2 = \text{cste}$,

$$\begin{cases} c_0 & = & \alpha_0\sqrt{gH}, \quad \text{with} \quad \alpha_0 = 1 + \dfrac{\varepsilon}{6} + \mathcal{O}(\varepsilon^2) \\ M_0(z) & = & 1 - \varepsilon\left[\dfrac{1}{2}\left(\dfrac{z}{H}\right)^2 + \left(\dfrac{z}{H}\right) + \dfrac{1}{3}\right] + \mathcal{O}(\varepsilon^2) \end{cases} \qquad \varepsilon = \frac{N^2 H}{g} = \mathcal{O}(10^{-2} - 10^{-4})$$

1 - The analysis framework

The vertical modes are orthonormal w.r.t. the scalar product $\langle f, g \rangle = \dfrac{1}{H}\displaystyle\int_{-H}^{0} f\,g\,dz$

$$u_q = \langle u, M_q \rangle \;,\; h_q = \frac{1}{\rho_0 g}\langle p, M_q \rangle$$

$$\left\{ \begin{array}{rcl} \partial_t u_q + g\,\partial_x h_q & = & 0 \\ \partial_t h_q + \dfrac{c_q^2}{g}\partial_x u_q & = & 0 \end{array} \right.$$

2 - Linear stability analysis : Shallow Water Equations

$$\left\{\begin{array}{rcl} \partial_t \tilde{h} + c\,\partial_x u & = & 0 \\ \partial_t u + c\,\partial_x \tilde{h} & = & 0 \end{array}\right. \qquad \tilde{h} = \frac{g}{c}\,h$$

von Neumann analysis : $\begin{array}{l} \tilde{h}(t)\,e^{ikx} \\ u(t)\,e^{ikx} \end{array} \longrightarrow \left\{\begin{array}{rcl} \partial_t \tilde{h} + c\,i\,k\,u & = & 0 \\ \partial_t u + c\,i\,k\,\tilde{h} & = & 0 \end{array}\right.$

2 - Linear stability analysis : Shallow Water Equations

$$\left\{\begin{array}{lcl} \partial_t \tilde{h} + c\,\partial_x u & = & 0 \\ \partial_t u + c\,\partial_x \tilde{h} & = & 0 \end{array}\right. \qquad \tilde{h} = \frac{g}{c}\,h$$

von Neumann analysis : $\begin{array}{l} \tilde{h}(t)\,e^{ikx} \\ u(t)\,e^{ikx} \end{array} \longrightarrow \left\{\begin{array}{lcl} \partial_t \tilde{h} + c\,i\,k\,u & = & 0 \\ \partial_t u + c\,i\,k\,\tilde{h} & = & 0 \end{array}\right.$

LFRA scheme with Brown-Campana pressure averaging :

$$\left\{\begin{array}{rcl} \tilde{h}^{n+1,*} & = & \tilde{h}^{n-1} - 2\Delta t\,c\,i\,k\,u^{n,*} \\ u^{n+1,*} & = & u^{n-1} - 2\Delta t\,c\,i\,k\,((1-2\gamma)\tilde{h}^{n,*} + \gamma(\tilde{h}^{n+1,*} + \tilde{h}^{n-1})) \\ \tilde{h}^{n} & = & (1-2\epsilon)\tilde{h}^{n,*} + \epsilon(\tilde{h}^{n+1,*} + \tilde{h}^{n-1}) \\ u^{n} & = & (1-2\epsilon)u^{n,*} + \epsilon(u^{n+1,*} + u^{n-1}) \end{array}\right.$$

3-time-level scheme, with a single rhs evaluation per time step

2 - Linear stability analysis : Shallow Water Equations

$$\left\{ \begin{array}{lcl} \partial_t \tilde{h} + c\,\partial_x u & = & 0 \\ \partial_t u + c\,\partial_x \tilde{h} & = & 0 \end{array} \right. \qquad \tilde{h} = \frac{g}{c}\,h$$

von Neumann analysis : $\begin{array}{l} \tilde{h}(t)\,e^{ikx} \\ u(t)\,e^{ikx} \end{array} \longrightarrow \left\{ \begin{array}{lcl} \partial_t \tilde{h} + c\,i\,k\,u & = & 0 \\ \partial_t u + c\,i\,k\,\tilde{h} & = & 0 \end{array} \right.$

RK3 scheme (Wicker Skamarock, 2002) with Forward-Backward feedback :

$$\left\{ \begin{array}{lcl} \tilde{h}^{n+1/3} & = & \tilde{h}^n - \frac{1}{3}\Delta t\,c\,i\,k\,u^n \\ u^{n+1/3} & = & u^n - \frac{1}{3}\Delta t\,c\,i\,k\,\tilde{h}^n \\ \tilde{h}^{n+1/2} & = & \tilde{h}^n - \frac{1}{2}\Delta t\,c\,i\,k\,u^{n+1/3} \\ u^{n+1/2} & = & u^n - \frac{1}{2}\Delta t\,c\,i\,k\,\tilde{h}^{n+1/3} \\ \tilde{h}^{n+1} & = & \tilde{h}^n - \Delta t\,c\,i\,k\,u^{n+1/2} \\ u^{n+1} & = & u^n - \Delta t\,c\,i\,k\left((1-2\gamma)\tilde{h}^{n+1/2} + \gamma(\tilde{h}^{n+1} + \tilde{h}^n)\right) \end{array} \right.$$

2-time-level scheme, with 3 rhs evaluations per time step

2 - Linear stability analysis : Shallow Water Equations

with $\mu = c\,k\,\Delta t$,

LFRA+BC scheme :

$$\begin{pmatrix} \tilde{h}^{n+1,*} \\ u^{n+1,*} \\ \tilde{h}^{n} \\ u^{n} \end{pmatrix} = \begin{bmatrix} A & B \\ \epsilon A + (1-2\epsilon) I_2 & \epsilon B + \epsilon I_2 \end{bmatrix} \begin{pmatrix} \tilde{h}^{n,*} \\ u^{n,*} \\ \tilde{h}^{n-1} \\ u^{n-1} \end{pmatrix}$$

$$A = \begin{bmatrix} 0 & -2i\mu \\ -2i\mu(1-2\gamma) & -4\gamma\mu^2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ -4\gamma i\mu & 1 \end{bmatrix}$$

RK3+FB scheme :

$$\begin{pmatrix} \tilde{h}^{n+1} \\ u^{n+1} \end{pmatrix} = \left[A + B * (I + \tfrac{1}{2} C * (I + \tfrac{1}{3} C))\right] \begin{pmatrix} \tilde{h}^{n} \\ u^{n} \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ -2i\gamma\mu & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -i\mu \\ -i\mu(1-2\gamma) & -\gamma\mu^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & -i\mu \\ -i\mu & 0 \end{bmatrix}$$

2 - Linear stability analysis : Shallow Water Equations

with $\mu = c\,k\,\Delta t$,

LFRA+BC scheme :

$$\begin{pmatrix} \tilde{h}^{n+1,*} \\ u^{n+1,*} \\ \tilde{h}^{n} \\ u^{n} \end{pmatrix} = \begin{bmatrix} A & B \\ \epsilon A + (1-2\epsilon) I_2 & \epsilon B + \epsilon I_2 \end{bmatrix} \begin{pmatrix} \tilde{h}^{n,*} \\ u^{n,*} \\ \tilde{h}^{n-1} \\ u^{n-1} \end{pmatrix}$$

$$A = \begin{bmatrix} 0 & -2i\mu \\ -2i\mu(1-2\gamma) & -4\gamma\mu^2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ -4\gamma i\mu & 1 \end{bmatrix}$$

RK3+FB scheme :

$$\begin{pmatrix} \tilde{h}^{n+1} \\ u^{n+1} \end{pmatrix} = \left[A + B * (I + \tfrac{1}{2} C * (I + \tfrac{1}{3} C))\right] \begin{pmatrix} \tilde{h}^{n} \\ u^{n} \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ -2i\gamma\mu & 1 \end{bmatrix}$$

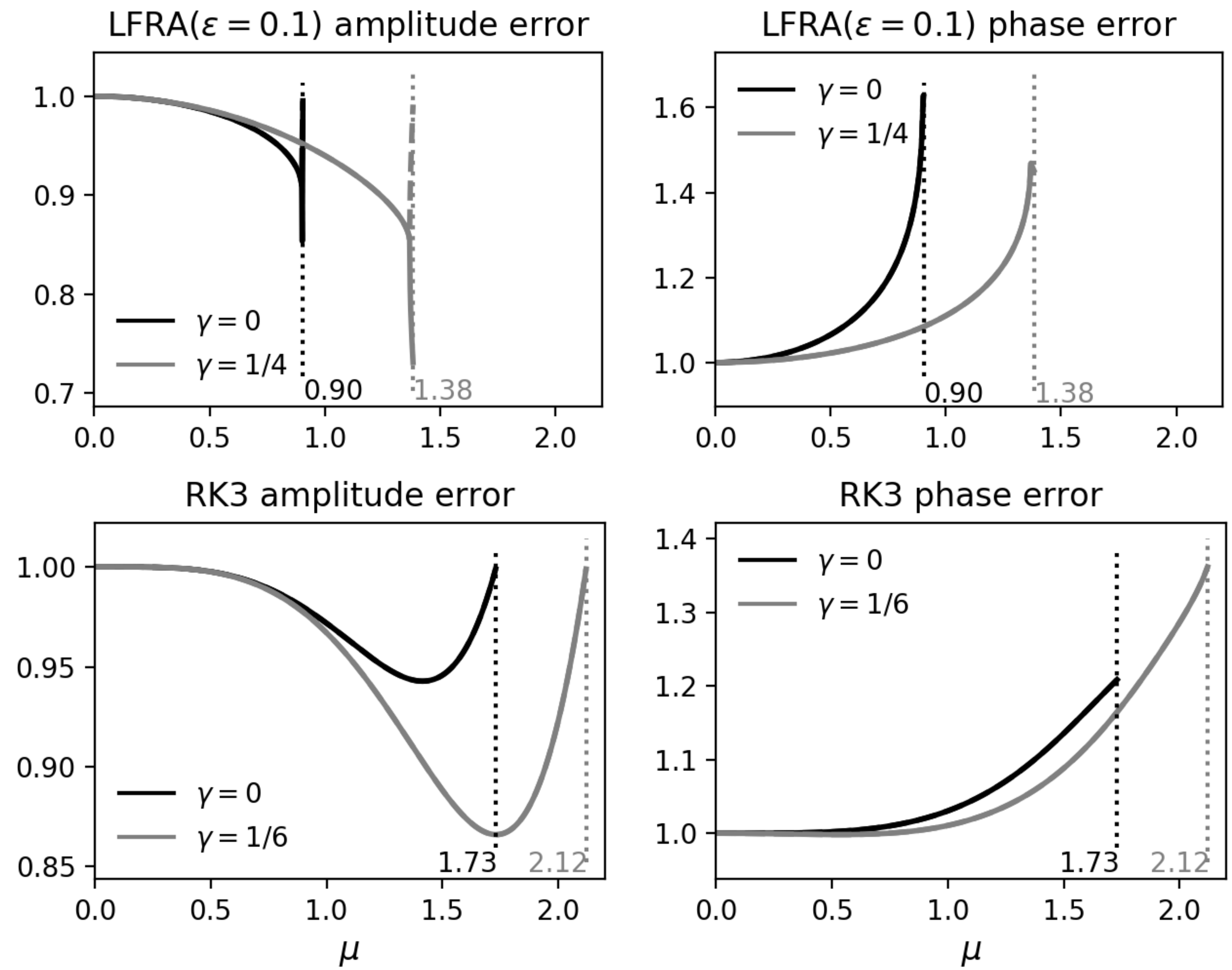
$$B = \begin{bmatrix} 0 & -i\mu \\ -i\mu(1-2\gamma) & -\gamma\mu^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & -i\mu \\ -i\mu & 0 \end{bmatrix}$$

Compute eigenvalues $\lambda^{\pm}(\mu)$ and compare with $e^{\pm i\mu}$:

$$E^{amp} = \frac{|\lambda^{\pm}(\mu)|}{|e^{\pm i\mu}|} = |\lambda^{\pm}(\mu)|, \quad E^{phase} = \frac{Arg(\lambda^{\pm}(\mu))}{Arg(e^{\pm i\mu})}$$

2 - Linear stability analysis : Shallow Water Equations



- both schemes dissipates and accelerates the gravity waves,
- BC and FB feedback extend the schemes stability : +53% and +22%,
- BC improves Leapfrog accuracy at all scales,
- FB feedback improves RK3 phase error but increases dissipation at small scales.

2 - Linear stability analysis : Shallow Water Equations

$$\left\{ \begin{array}{lcl} \partial_t \tilde{h} + u_0 \, \partial_x \tilde{h} + c \, \partial_x u & = & 0 \\ \partial_t u + u_0 \, \partial_x u + c \, \partial_x \tilde{h} & = & 0 \end{array} \right.$$

von Neumann analysis : $\begin{array}{l} \tilde{h}(t) \, e^{ikx} \\ u(t) \, e^{ikx} \end{array} \longrightarrow \left\{ \begin{array}{lcl} \partial_t \tilde{h} + u_0 \, i \, k \, \tilde{h} + c \, i \, k \, u & = & 0 \\ \partial_t u + u_0 \, i \, k \, u + c \, i \, k \, \tilde{h} & = & 0 \end{array} \right.$

2 - Linear stability analysis : Shallow Water Equations

$$\left\{ \begin{array}{lcl} \partial_t \tilde{h} + u_0 \, \partial_x \tilde{h} + c \, \partial_x u & = & 0 \\ \partial_t u + u_0 \, \partial_x u + c \, \partial_x \tilde{h} & = & 0 \end{array} \right.$$

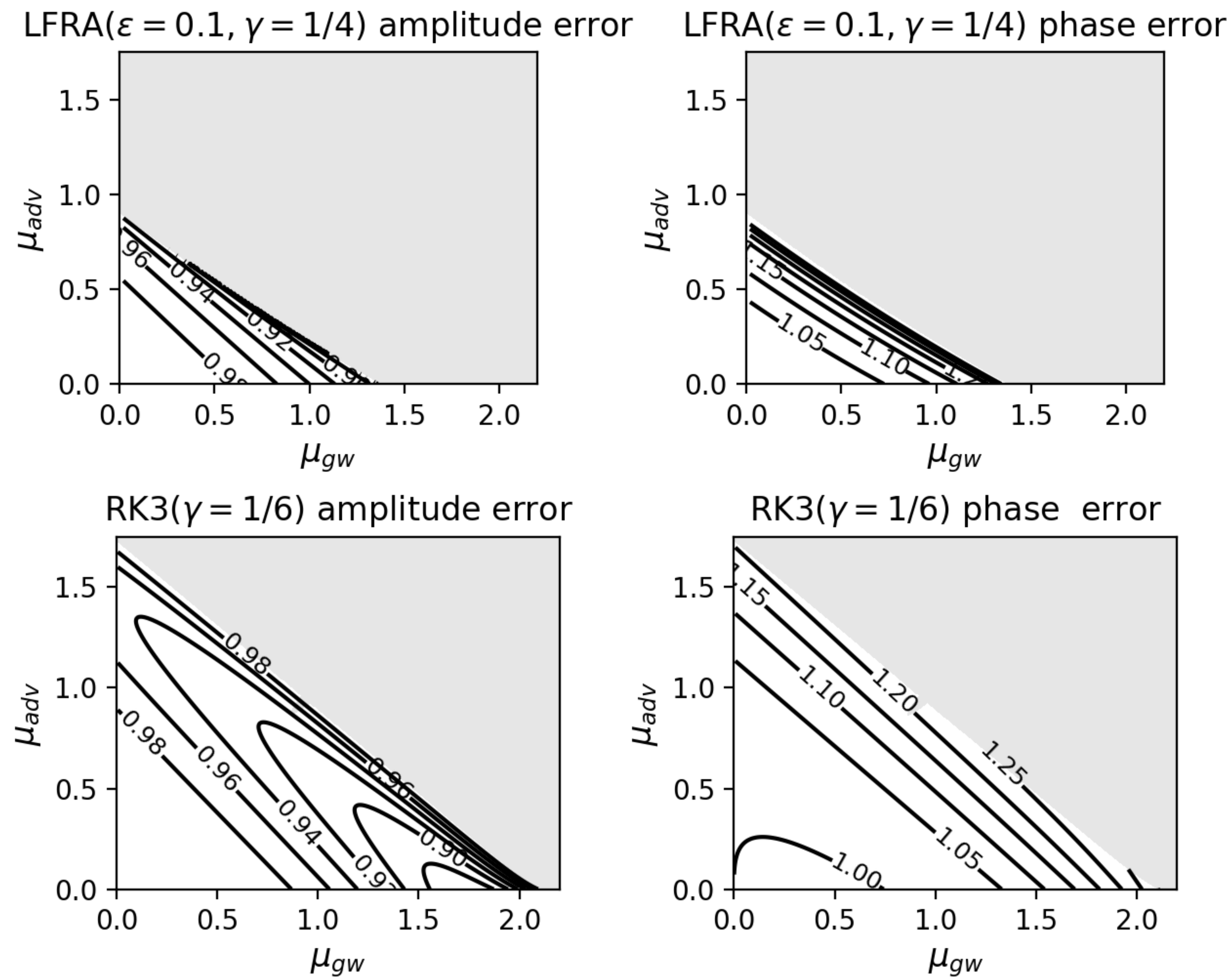
von Neumann analysis : $\begin{array}{l} \tilde{h}(t) \, e^{ikx} \\ u(t) \, e^{ikx} \end{array} \longrightarrow \left\{ \begin{array}{lcl} \partial_t \tilde{h} + u_0 \, i \, k \, \tilde{h} + c \, i \, k \, u & = & 0 \\ \partial_t u + u_0 \, i \, k \, u + c \, i \, k \, \tilde{h} & = & 0 \end{array} \right.$

Apply LFRA+BC and RK3+FB schemes,

Write in matrix form, which now depends on $\mu_{gw} = c \, k \, \Delta t, \;\; \mu_{adv} = u_0 \, k \, \Delta t$

Compute eigenvalues $\lambda^{\pm}(\mu_{gw}, \mu_{adv})$ and compare with $e^{-i(\mu_{adv} \mp \mu_{gw})}$

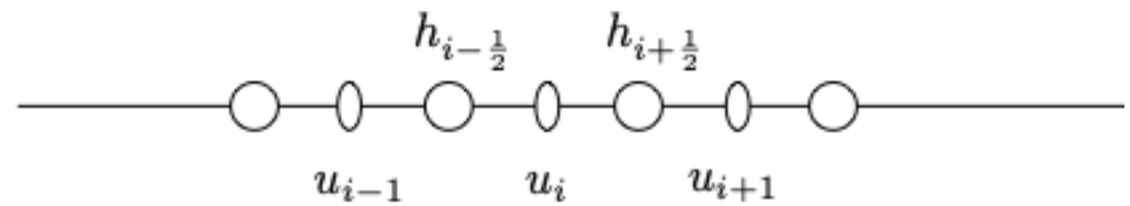
2 - Linear stability analysis : Shallow Water Equations



- for pure GW, we recover previous results with BC and FB feedback at play,
- the pure advection has stability properties identical to pure GW without BC and FB feedback,
- in between, smooth transition

2 - Linear stability analysis : Shallow Water Equations

$$\left\{\begin{array}{rcl} \partial_t \tilde{h}_{i-\frac{1}{2}} + u_0\,\mathcal{A}[\tilde{h}]_{i-\frac{1}{2}} + c\,\mathcal{G}[u]_{i-\frac{1}{2}} & = & 0 \\ \partial_t u_i + u_0\,\mathcal{A}[u]_i + c\,\mathcal{G}[\tilde{h}]_i & = & 0 \end{array}\right.$$



von Neumann analysis :

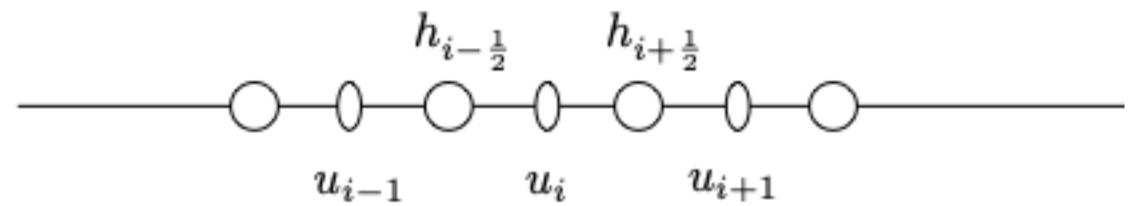
$$\begin{array}{r} i\,\mu_{gw} \to \mathcal{S}(\mathcal{G})\,\mu_{gw} \\ i\,\mu_{adv} \to \mathcal{S}(\mathcal{A})\,\mu_{adv} \end{array} \qquad \mu_{gw} = c\frac{\Delta t}{\Delta x} \quad \mu_{adv} = u_0\frac{\Delta t}{\Delta x}$$

$$\begin{array}{rcl} \mathcal{S}(\mathcal{G}^{C2}) & = & 2i\sin(\theta/2) \\ \mathcal{S}(\mathcal{A}^{C2}) & = & i\sin\theta \\ \mathcal{S}(\mathcal{A}^{C4}) & = & i(8\sin\theta - \sin 2\theta)/6 \\ \mathcal{S}(\mathcal{A}^{UP3}) & = & i(8\sin\theta - \sin 2\theta)/6 + 8\sin(\theta/2)^4/6 \\ \mathcal{S}(\mathcal{A}^{UP5}) & = & i(\sin 3\theta - 9\sin 2\theta + 45\sin\theta)/30 - (\cos 3\theta - 6\cos 2\theta + 15\cos\theta - 10)/30 \end{array}$$

$$\theta = k\Delta x$$

2 - Linear stability analysis : Shallow Water Equations

$$\begin{cases} \partial_t \tilde{h}_{i-\frac{1}{2}} + u_0\, \mathcal{A}[\tilde{h}]_{i-\frac{1}{2}} + c\, \mathcal{G}[u]_{i-\frac{1}{2}} & = & 0 \\ \partial_t u_i + u_0\, \mathcal{A}[u]_i + c\, \mathcal{G}[\tilde{h}]_i & = & 0 \end{cases}$$



von Neumann analysis :

$$\begin{aligned} i\, \mu_{gw} &\rightarrow \mathcal{S}(\mathcal{G})\, \mu_{gw} \\ i\, \mu_{adv} &\rightarrow \mathcal{S}(\mathcal{A})\, \mu_{adv} \end{aligned} \qquad \mu_{gw} = c\frac{\Delta t}{\Delta x} \qquad \mu_{adv} = u_0 \frac{\Delta t}{\Delta x}$$

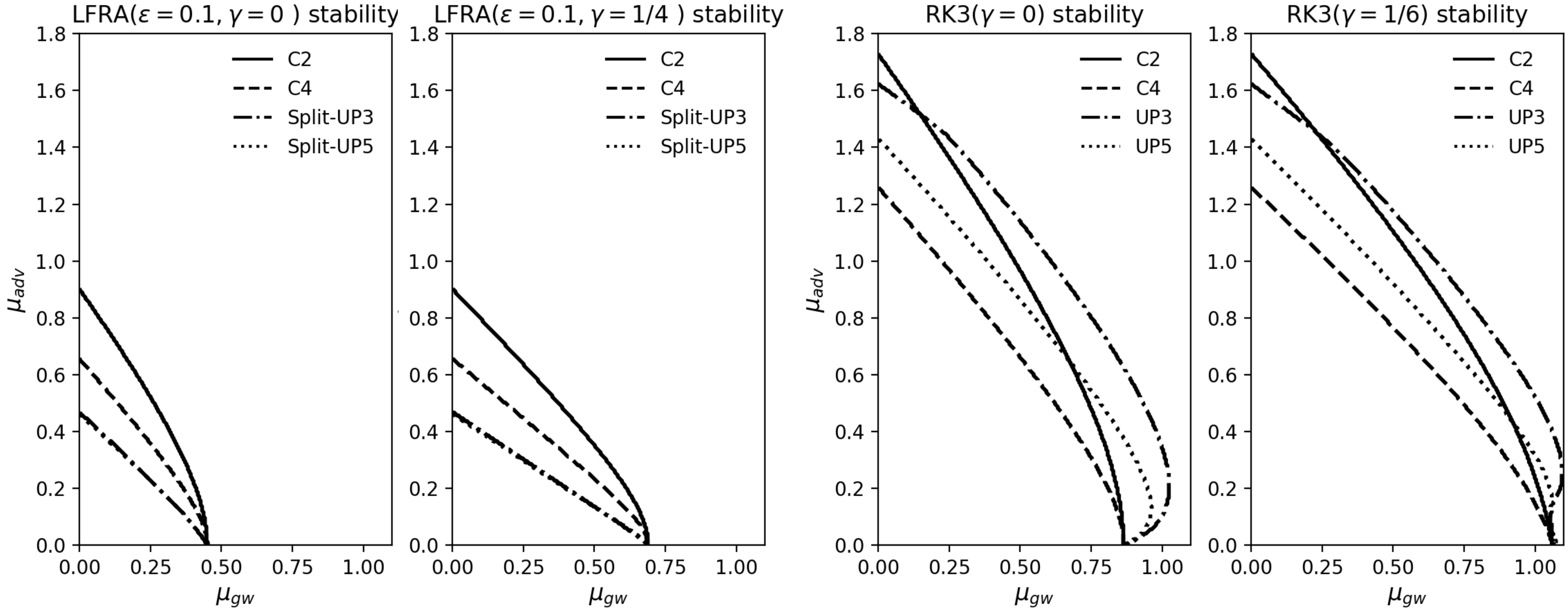
$$\begin{aligned} \mathcal{S}(\mathcal{G}^{C2}) &= 2i \sin(\theta/2) \\ \mathcal{S}(\mathcal{A}^{C2}) &= i \sin\theta \\ \mathcal{S}(\mathcal{A}^{C4}) &= i(8\sin\theta - \sin 2\theta)/6 \\ \mathcal{S}(\mathcal{A}^{UP3}) &= i(8\sin\theta - \sin 2\theta)/6 + 8\sin(\theta/2)^4/6 \\ \mathcal{S}(\mathcal{A}^{UP5}) &= i(\sin 3\theta - 9\sin 2\theta + 45\sin\theta)/30 - (\cos 3\theta - 6\cos 2\theta + 15\cos\theta - 10)/30 \end{aligned}$$

$$\theta = k\Delta x$$

Apply LFRA+BC and RK3+FB schemes, write in matrix form, compute $\lambda^{\pm}(\theta, \mu_{gw}, \mu_{adv})$

2 - Linear stability analysis : Shallow Water Equations

$$\max_{0\leq\theta\leq\pi} |\lambda^{\pm}(\theta, \mu_{gw}, \mu_{adv})| = 1$$



- for pure GW, the stability limit is half the limit of the semi-discrete analysis,
- the stability of GW and advection is very sensitive to advection scheme :
  - for both LFRA and RK3, the use of C4 decreases the stability limit w.r.t C2,
  - for LFRA, the stability limit with Split-UP3 and Split-UP5 is even smaller,
  - for RK3, there is a marked ‘overshoot’ of stability with UP3 and UP5.

3 - Linear stability analysis : Hydrostatic Primitive Equations

Usual practice in split ocean models : do not integrate the true barotropic mode but

$$\left\{ \begin{array}{rcl} \partial_t \eta + H \partial_x \overline{u} & = & 0 \\ \partial_t \overline{u} + g \partial_x \eta & = & -\dfrac{1}{\rho_0} \partial_x \left(\dfrac{1}{H} \displaystyle\int_{-H}^{0} p_h \, dz \right) \end{array} \right.$$

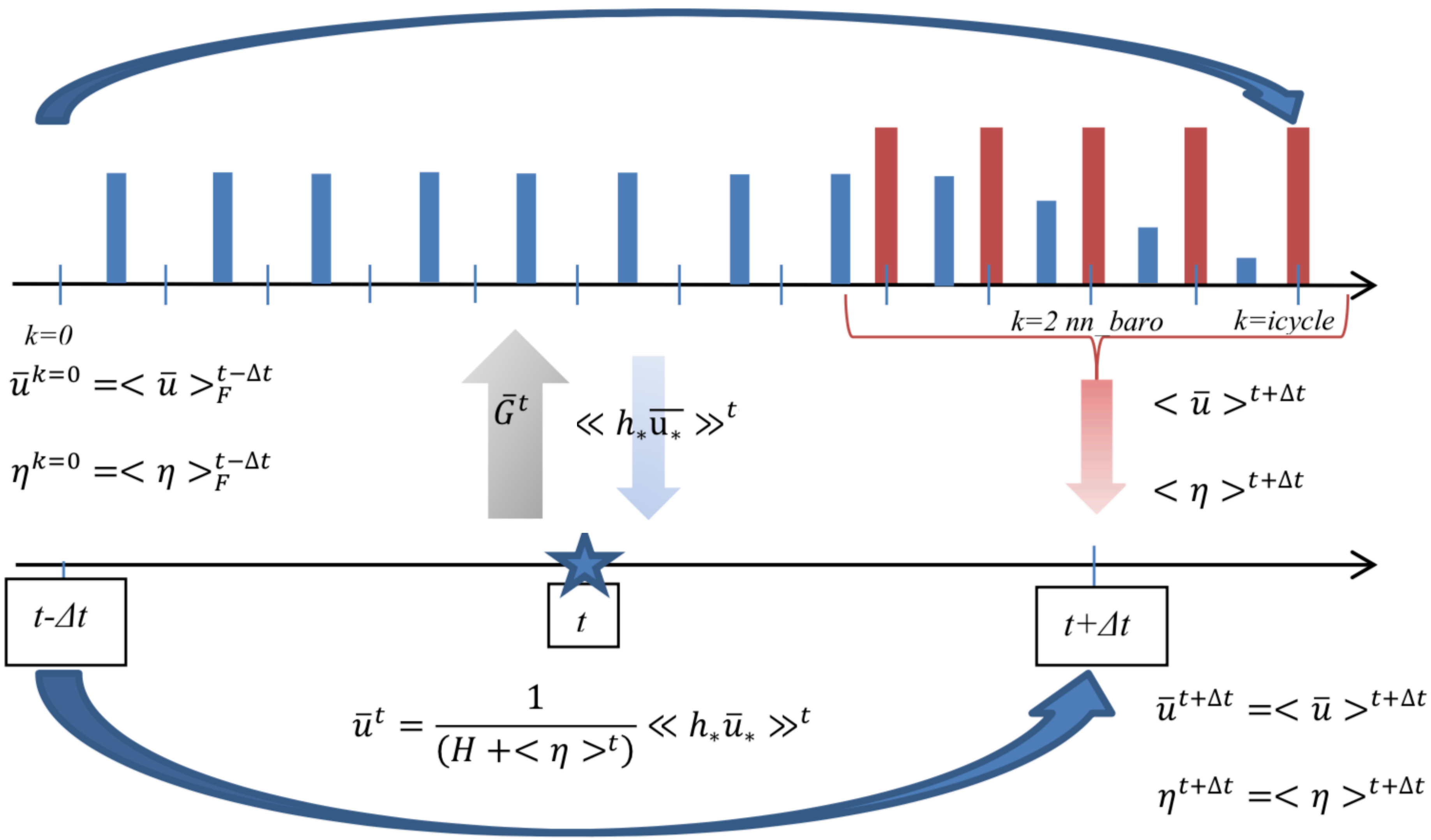
- the red term is a "slow" term absent from the normal mode analysis,
- in practice, this term is kept frozen during the barotropic integration,
- but it contains fast contributions -> source of instability for the barotropic integration

- barotropic/baroclinic corrections have to be done to ensure their compatibility

- dissipation within the barotropic mode is necessary to stabilize the integration
- dissipation can be introduced through averaging filter or dissipative time stepping

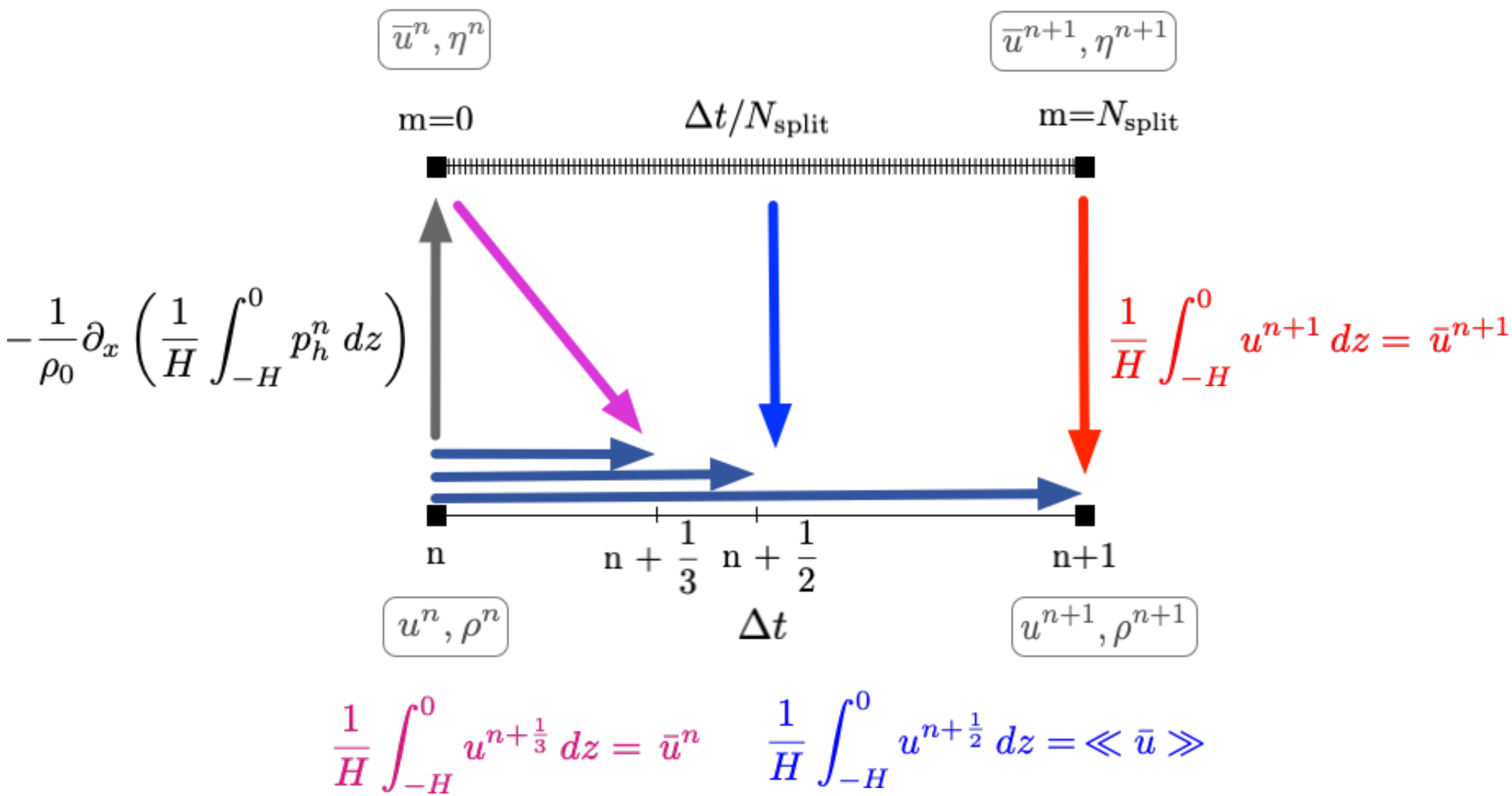
3 - Linear stability analysis : Hydrostatic Primitive Equations

NEMO LFRA-based mode-splitting algorithm (cf NEMO Book)



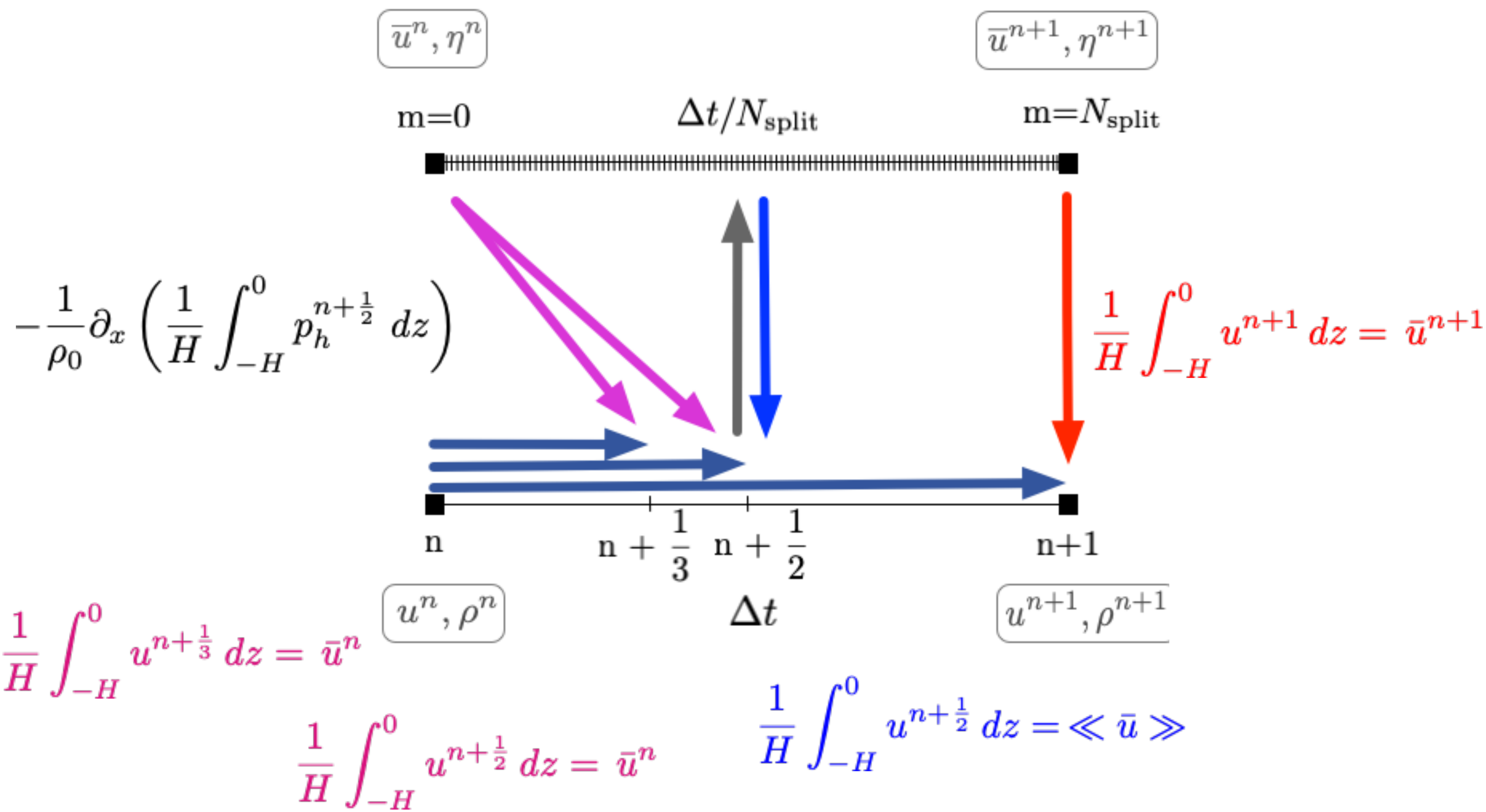
3 - Linear stability analysis : Hydrostatic Primitive Equations

RK3-based mode-splitting algorithm : 1st stage coupled



3 - Linear stability analysis : Hydrostatic Primitive Equations

RK3-based mode-splitting algorithm : 3rd stage coupled



3 - Linear stability analysis : Hydrostatic Primitive Equations

Demange etal. 2019 proposed a framework to study the stability of mode-splitting algorithms

- express in terms of normal modes the components of the splitting algorithm
- build the step-multiplier matrix for each vertical mode

$$\begin{pmatrix} h_q \\ u_q \end{pmatrix}^{n+1} = A_q^{\mathrm{3d}} \begin{pmatrix} h_q \\ u_q \end{pmatrix}^{n} + C_q \sum_p V_p \left(A_p^{\mathrm{2d}} - A_p^{\mathrm{3d}}\right) \begin{pmatrix} h_p \\ u_p \end{pmatrix}^{n}$$

3 - Linear stability analysis : Hydrostatic Primitive Equations

Demange etal. 2019 proposed a framework to study the stability of mode-splitting algorithms

- express in terms of normal modes the components of the splitting algorithm
- build the step-multiplier matrix for each vertical mode
- restricting attention to the barotropic and 1st baroclinic modes

$$\begin{pmatrix} h_0 \\ u_0 \\ h_1 \\ u_1 \end{pmatrix}^{n+1} = \begin{bmatrix} A_0^{3\mathrm{d}} + C_0 V_0 \left(A_0^{2\mathrm{d}} - A_0^{3\mathrm{d}} \right) & C_0 V_1 \left(A_1^{2\mathrm{d}} - A_1^{3\mathrm{d}} \right) \\ C_1 V_0 \left(A_0^{2\mathrm{d}} - A_0^{3\mathrm{d}} \right) & A_1^{3\mathrm{d}} + C_1 V_1 \left(A_1^{2\mathrm{d}} - A_1^{3\mathrm{d}} \right) \end{bmatrix} \begin{pmatrix} h_0 \\ u_0 \\ h_1 \\ u_1 \end{pmatrix}^{n}$$

3 - Linear stability analysis : Hydrostatic Primitive Equations

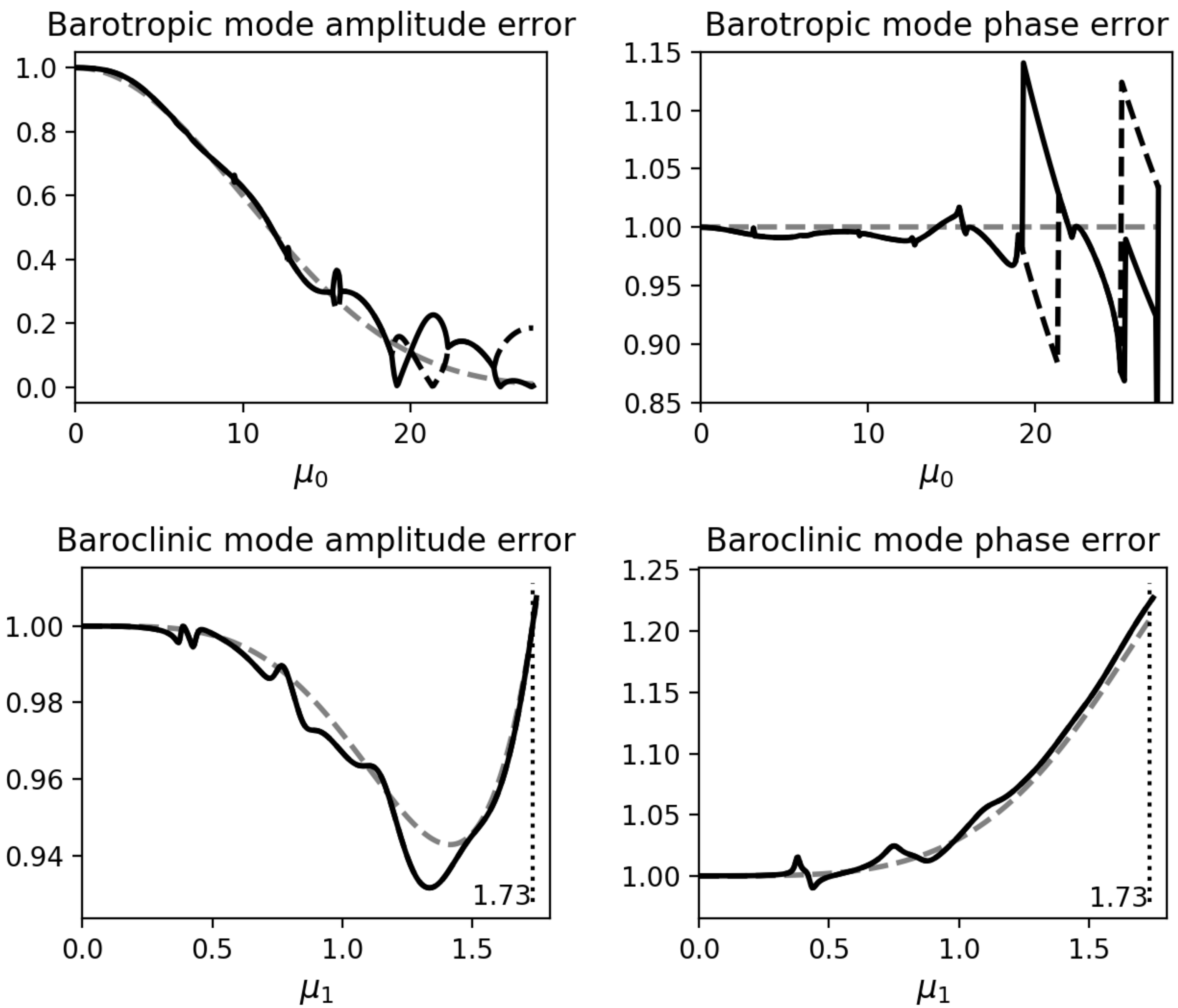
Demange etal. 2019 proposed a framework to study the stability of mode-splitting algorithms

- express in terms of normal modes the components of the splitting algorithm
- build the step-multiplier matrix for each vertical mode
- restricting attention to the barotropic and 1st baroclinic modes

$$\begin{pmatrix} h_0 \\ u_0 \\ h_1 \\ u_1 \end{pmatrix}^{n+1} = \begin{bmatrix} A_0^{3\mathrm{d}} + C_0 V_0 \left(A_0^{2\mathrm{d}} - A_0^{3\mathrm{d}}\right) & C_0 V_1 \left(A_1^{2\mathrm{d}} - A_1^{3\mathrm{d}}\right) \\ C_1 V_0 \left(A_0^{2\mathrm{d}} - A_0^{3\mathrm{d}}\right) & A_1^{3\mathrm{d}} + C_1 V_1 \left(A_1^{2\mathrm{d}} - A_1^{3\mathrm{d}}\right) \end{bmatrix} \begin{pmatrix} h_0 \\ u_0 \\ h_1 \\ u_1 \end{pmatrix}^{n}$$

- compute the eigenvalues for the split-RK3 :
  - with mode-splitting at the 1st stage,
  - correction at n+1/3 with transport at n,
  - and the 2d integration being integrated without dispersion errors but the dissipation of the dissipative Forward-Backward scheme (cf Demange etal. 2019)

3 - Linear stability analysis : Hydrostatic Primitive Equations



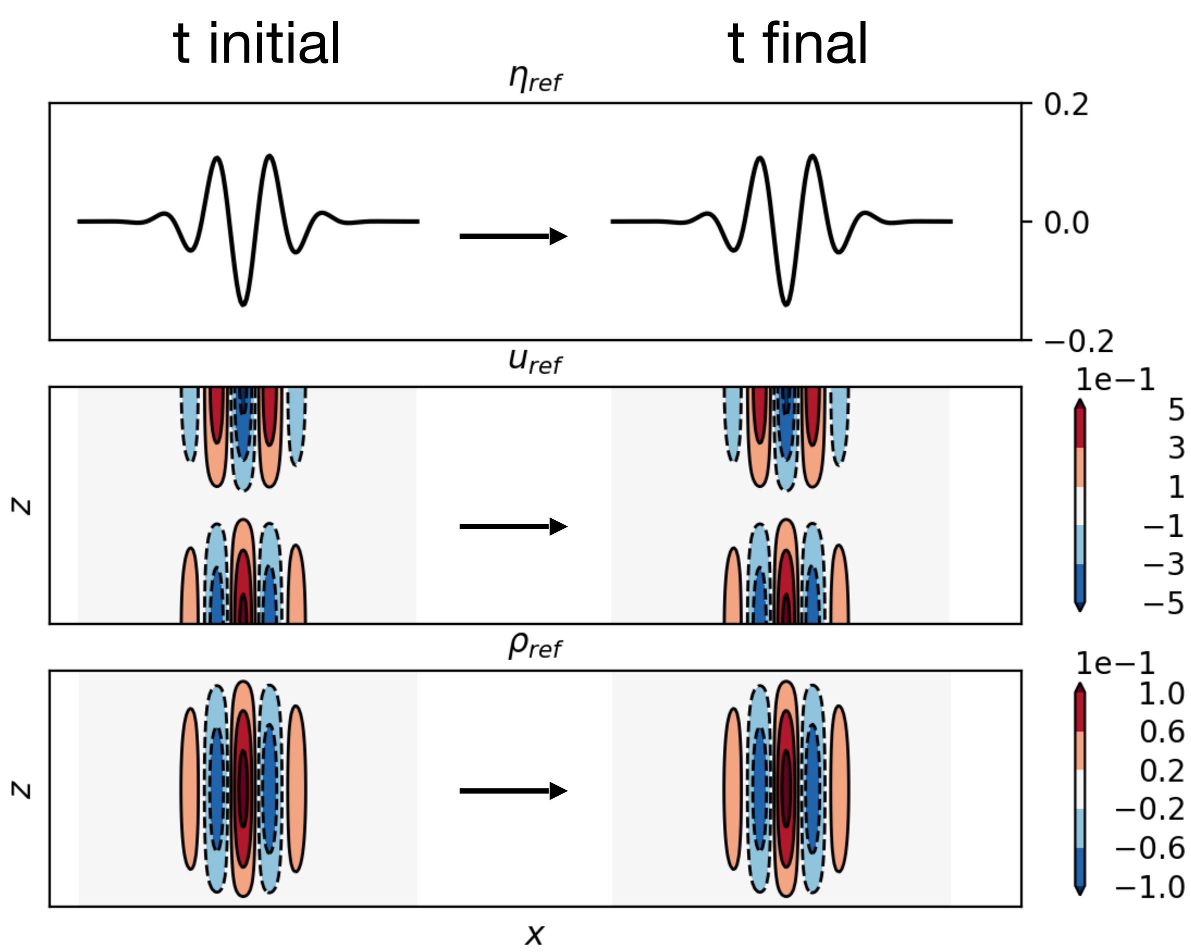
N = 10-2 s-1
H = 4000 m
g = 9.81 m.s-2

c0/c1 = 15.6
Nsplit = 20

theta = 0.2

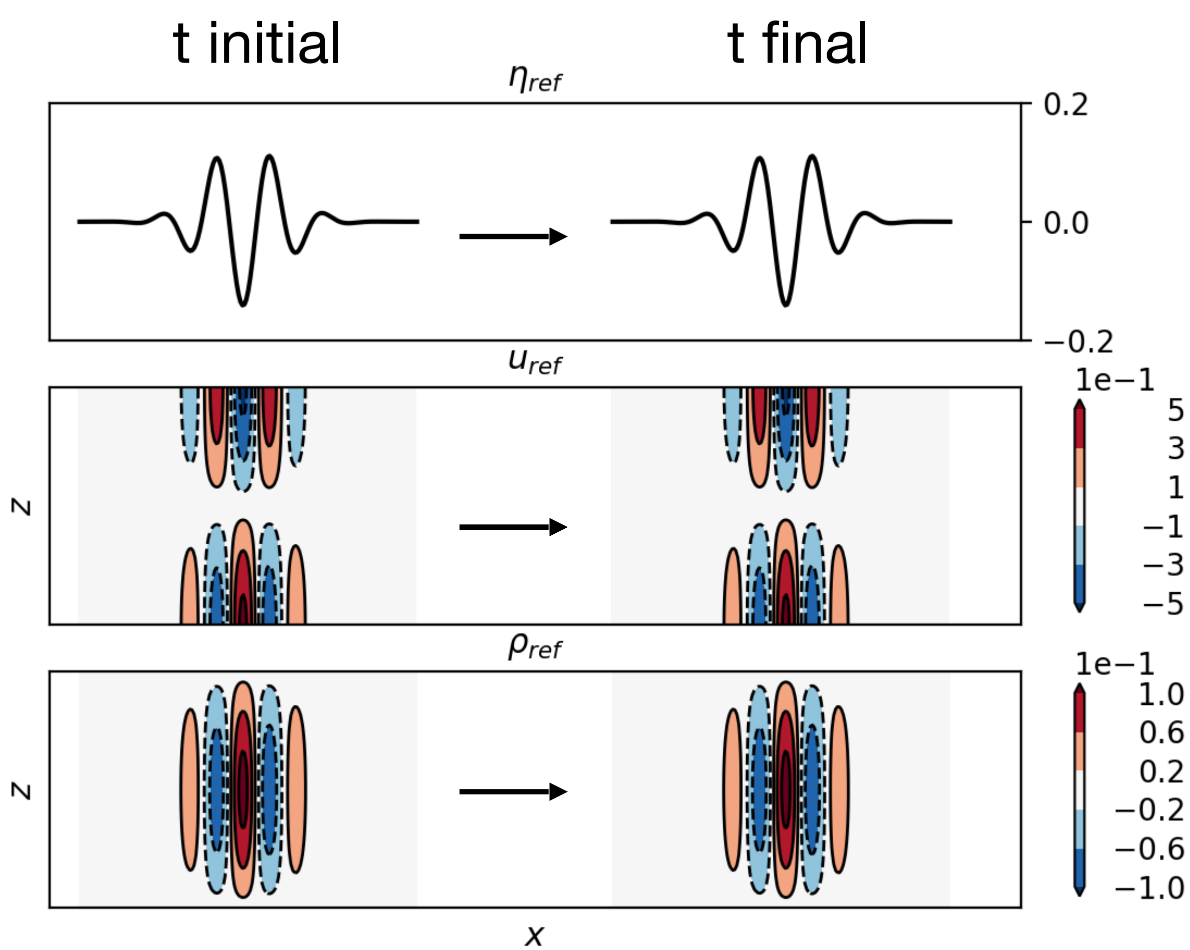
the baroclinic time step has not to be reduced from the stability limit of the RK3 scheme

4 - An idealized test-case

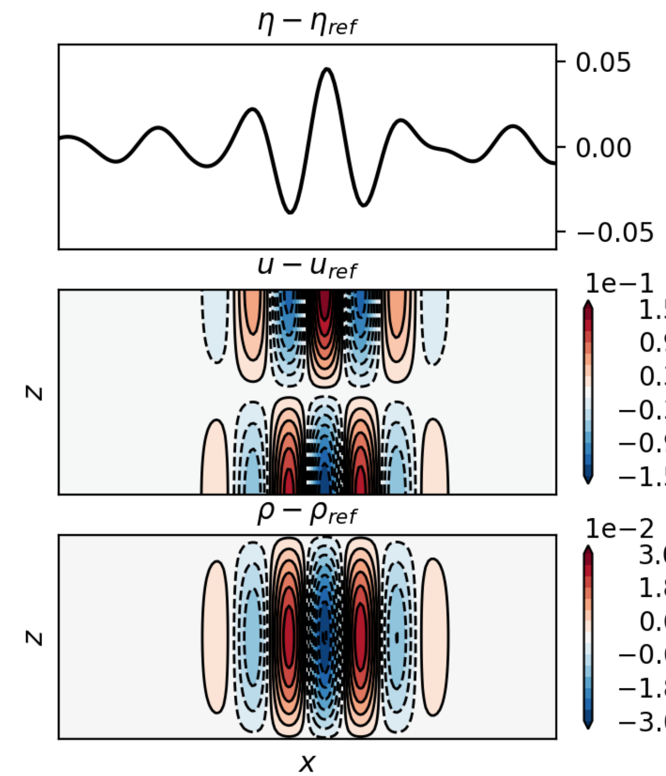


N = 2.10-3 s-1
H = 4000 m
g = 9.81 m.s-2
c0/c1 = 77.8

4 - An idealized test-case

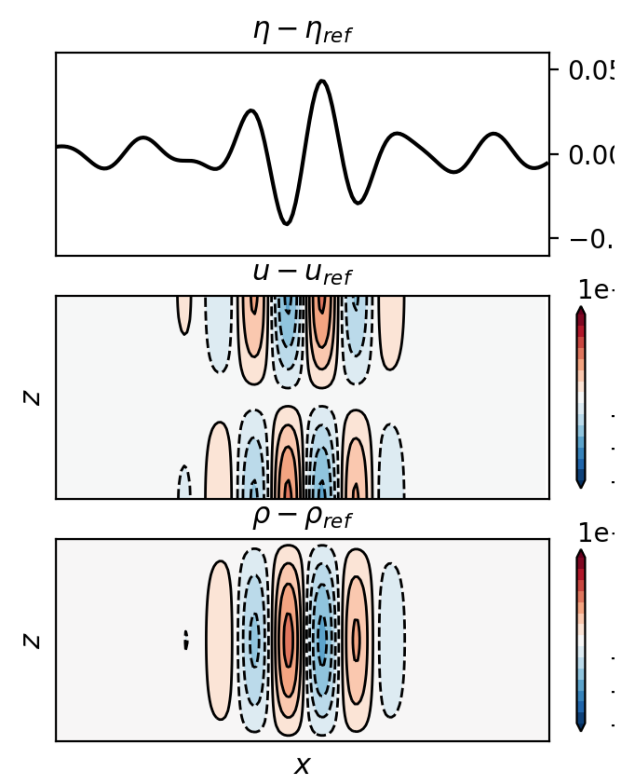


N = 2.10-3 s-1
H = 4000 m
g = 9.81 m.s-2
c0/c1 = 77.8



split-LFRA

dt = 627s
Nsplit = 80
theta = 0.2



split-RK3

dt = 1254s
Nsplit = 80
theta = 0.2

Preliminary conclusions

- the stability analysis of the fully discrete combined GW-advection problem confirms that RK3+FB is an attractive alternative to LFRA,
- the stability analysis of the mode-splitting algorithm indicates that a dissipative 2d integration scheme is able to stabilise the 1st-stage coupled split-RK3 prototype,
- first numerical experiments with the 1st-stage coupled split-RK3 prototype show benefits in term of accuracy/stability w.r.t the split-LFRA