M_2 internal-tide generation in STORMTIDE2

Jin-Song von Storch & Zhuhua Li

M_2 internal-tide generation in STORMTIDE2

Jin-Song von Storch & Zhuhua Li

- How to accurately estimate internal-tide generation?
  - with focus on the understanding
- How is M_2 internal-tide generated in STORMTIDE2?
  - MPIOM, 0.1° resolution
  - driven by the full lunisolar tidal potential + 6 hourly NCEP forcing

Known: Two concepts of IT-generation

Known: Two concepts of IT-generation

Based on work done by a form drag
(Bell, 1975, Llewellyn Smith & Young 2002, Nycander 2005, Vic, et al. 2019,…):

$$\mathscr{P} = -\mathbf{U}_H \cdot \left(p^i_{-d} \nabla d\right)$$

p^i_{-d} : internal-tide pressure at $z = -d$

$\mathbf{U}_H = (U, V)$: the tidal velocity

∇: horizontal differential operator

Known: Two concepts of IT-generation

Based on work done by a form drag
(Bell, 1975, Llewellyn Smith & Young 2002, Nycander 2005, Vic, et al. 2019,...):

$$\mathscr{P} = -\mathbf{U}_H \cdot \left(p^i_{-d} \nabla d\right)$$

p^i_{-d} : internal-tide pressure at $z = -d$

$\mathbf{U}_H = (U, V)$: the tidal velocity

∇: horizontal differential operator

Based on conversion from barotropic energy to baroclinic energy
(Niwa & Hibiya, 2004, Kang & Fringer, 2012, Müller, 2013,...):

$$\mathscr{C} = \overline{g\rho' W}$$

$$\overline{(\cdot)} = \int_{-d}^{\eta} \cdot \, dz$$

$$\nabla_H \cdot \mathbf{U}_H + \frac{\partial W}{\partial z} = 0$$

Known: Two concepts of IT-generation

Based on work done by a form drag
(Bell, 1975, Llewellyn Smith & Young 2002, Nycander 2005, Vic, et al. 2019,…):

$$\mathscr{P} = -\mathbf{U}_H \cdot \left(p^i_{-d} \nabla d\right)$$

p^i_{-d}: internal-tide pressure at $z = -d$

$\mathbf{U}_H = (U, V)$: the tidal velocity

∇: horizontal differential operator

Based on conversion from barotropic energy to baroclinic energy
(Niwa & Hibiya, 2004, Kang & Fringer, 2012, Müller, 2013,…):

$$\mathscr{C} = \overline{g\rho' W}$$

$$\overline{(\cdot)} = \int_{-d}^{\eta} \cdot \, dz$$

$$\nabla_H \cdot \mathbf{U}_H + \frac{\partial W}{\partial z} = 0$$

The two concepts should lead to the same result!?

Known: different ways to determine p^i

Known: different ways to determine p^i

- using semi-analytical solution, assuming weak topography and small tidal excursion
- decomposing the full (observed or simulated) pressure p to get p^i

Known: different ways to determine p^i

- using semi-analytical solution, assuming weak topography and small tidal excursion
- decomposing the full (observed or simulated) pressure p to get p^i

- Kelly et al. 2010 assume $p = p^i + p^s$ and summarize 5 different methods:

$p^s = P + \hat{p},\ p^i = p' - \hat{p}$ where $P = \frac{1}{h}\overline{p},\ p' = p - P$

Known: different ways to determine p^i

- using semi-analytical solution, assuming weak topography and small tidal excursion
- decomposing the full (observed or simulated) pressure p to get p^i

- Kelly et al. 2010 assume $p = p^i + p^s$ and summarize 5 different methods:

$p^s = P + \hat{p},\ p^i = p' - \hat{p}$ where $P = \frac{1}{h}\overline{p},\ p' = p - P$

- The method with $\hat{p} = 0$ „*provides physical interpretable conversion but also contains an error*“

Known: different ways to determine p^i

- using semi-analytical solution, assuming weak topography and small tidal excursion
- decomposing the full (observed or simulated) pressure p to get p^i

- Kelly et al. 2010 assume $p = p^i + p^s$ and summarize 5 different methods:

$$p^s = P + \hat{p},\ p^i = p' - \hat{p} \quad \text{where } P = \frac{1}{h}\overline{p},\ p' = p - P$$

- The method with $\hat{p} = 0$ „*provides physical interpretable conversion but also contains an error*“

- They suggest to use $\hat{p}(x, y, z)$ that describes the isophycnal heaving due to free surface movement → $\mathscr{P}$ is the IT generation

Do energy considerations always lead to the same result?

Do energy considerations always lead to the same result?

- General procedure used in an energy consideration (Simmons et al. 2004, Kang & Fringer 2012, Kelly et al. 2010),…):
    1. Derive the equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$
    2. Derive the (kinetic) energy equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$
    3. Depth-integrate the energy equations

Do energy considerations always lead to the same result?

- General procedure used in an energy consideration (Simmons et al. 2004, Kang & Fringer 2012, Kelly et al. 2010),…):
    1. Derive the equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $p'_\eta \nabla \eta + p'_{-d} \nabla d$
    2. Derive the (kinetic) energy equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$
    3. Depth-integrate the energy equations

Do energy considerations always lead to the same result?

- General procedure used in an energy consideration (Simmons et al. 2004, Kang & Fringer 2012, Kelly et al. 2010),…):

1. Derive the equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $p'_\eta \nabla \eta + p'_{-d} \nabla d$
2. Derive the (kinetic) energy equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $(\mathbf{U}_H \cdot (p'_\eta \nabla \eta) + \mathbf{U}_H \cdot (p'_{-d} \nabla d))/H$
3. Depth-integrate the energy equations

Do energy considerations always lead to the same result?

- General procedure used in an energy consideration (Simmons et al. 2004, Kang & Fringer 2012, Kelly et al. 2010),…):

1. Derive the equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $p'_\eta \nabla \eta + p'_{-d} \nabla d$
2. Derive the (kinetic) energy equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $(\mathbf{U}_H \cdot (p'_\eta \nabla \eta) + \mathbf{U}_H \cdot (p'_{-d} \nabla d))/H$
3. Depth-integrate the energy equations → $\overline{\frac{\partial p' W}{\partial z}} - \overline{\frac{\partial p' W}{\partial z}}$

$$= \overline{p'} \frac{\partial W}{\partial z} - \overline{g \rho' W} - (p' W)_\eta + (p' W)_{-d}$$

Do energy considerations always lead to the same result?

- General procedure used in an energy consideration (Simmons et al. 2004, Kang & Fringer 2012, Kelly et al. 2010),...):

  1. Derive the equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $p'_\eta \nabla \eta + p'_{-d} \nabla d$
  2. Derive the (kinetic) energy equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $(\mathbf{U}_H \cdot (p'_\eta \nabla \eta) + \mathbf{U}_H \cdot (p'_{-d} \nabla d))/H$
  3. Depth-integrate the energy equations → $\overline{\dfrac{\partial p'W}{\partial z}} - \overline{\dfrac{\partial p'W}{\partial z}}$

$$= \overline{p'}\frac{\partial W}{\partial z} - \overline{g\rho' W} - (p'W)_\eta + (p'W)_{-d}$$

- When decomposing $w = W + w'$ following Kang and Fringer 2012:

$$\nabla_H \cdot \mathbf{U} + \frac{\partial W}{\partial z} = 0$$

$$W_\eta = \frac{\partial \eta}{\partial t} + \mathbf{U}_H \cdot \nabla \eta, \quad \text{at } z = \eta$$

$$W_{-d} = -\mathbf{U}_H \cdot \nabla_H d, \quad \text{at } z = -d$$

Do energy considerations always lead to the same result?

- General procedure used in an energy consideration (Simmons et al. 2004, Kang & Fringer 2012, Kelly et al. 2010),…):

    1. Derive the equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $p'_\eta \nabla\eta + p'_{-d}\nabla d$
    2. Derive the (kinetic) energy equations of $\mathbf{U}_H$ and $\mathbf{u}'_H$ → $(\mathbf{U}_H \cdot (p'_\eta \nabla\eta) + \mathbf{U}_H \cdot (p'_{-d}\nabla d))/H$
    3. Depth-integrate the energy equations → $\overline{\frac{\partial p'W}{\partial z}} - \overline{\frac{\partial p'W}{\partial z}}$

$$= \overline{p'\frac{\partial W}{\partial z}} - \overline{g\rho' W} - (p'W)_\eta + (p'W)_{-d}$$

- When decomposing $w = W + w'$ following Kang and Fringer 2012:

$$\nabla_H \cdot \mathbf{U} + \frac{\partial W}{\partial z} = 0$$

$$W_\eta = \frac{\partial \eta}{\partial t} + \mathbf{U}_H \cdot \nabla\eta, \quad \text{at } z = \eta$$

$$W_{-d} = -\mathbf{U}_H \cdot \nabla_H d, \quad \text{at } z = -d$$

→

- **form drags cancel $(p'W)_\eta$ and $(p'W)_{-d}$**
- **$\mathscr{C} = \overline{g\rho' W}$ is the IT generation**
- **No cancelation when using any other decomposition (e.g. in Kelly et al. 2010)**

Pressure decomposition

Pressure decomposition

Using the density decomposition $\rho = \rho_\circ + \rho_b(x, y, z) + \rho'(x, y, z, t)$, the hydrostatic pressure is decomposed as $p = p_\circ + p_b(x, y, z) + p'(x, y, z, t)$ with

$$\frac{\partial \mathbf{u}_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla p' + \cdots$$

Pressure decomposition

Using the density decomposition $\rho = \rho_\circ + \rho_b(x,y,z) + \rho'(x,y,z,t)$, the hydrostatic pressure is decomposed as $p = p_\circ + p_b(x,y,z) + p'(x,y,z,t)$ with

$$\frac{\partial \mathbf{u}_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla p' + \cdots$$

One has, with $\overline{\overline{(\cdot)}} = \overline{(\cdot)}/H$ and $p'' = p' - \overline{\overline{p'}}$,

$$\frac{\partial \mathbf{U}_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla \overline{\overline{p'}} + \underbrace{\frac{1}{\rho_0 H}\left(p''_\eta \nabla \eta + p''_d \nabla d\right)}_{\text{coupling terms}} + \cdots$$

and

$$\frac{\partial \mathbf{u}'_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla p'' \underbrace{- \frac{1}{\rho_0 H}\left(p''_\eta \nabla \eta + p''_d \nabla d\right)}_{\text{coupling terms}} + \cdots .$$

Pressure decomposition

Using the density decomposition $\rho = \rho_\circ + \rho_b(x,y,z) + \rho'(x,y,z,t)$, the hydrostatic pressure is decomposed as $p = p_\circ + p_b(x,y,z) + p'(x,y,z,t)$ with

$$\frac{\partial \mathbf{u}_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla p' + \cdots$$

One has, with $\overline{\overline{(\,\cdot\,)}} = \overline{(\,\cdot\,)}/H$ and $p'' = p' - \overline{\overline{p'}}$,

$$\frac{\partial \mathbf{U}_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla \overline{\overline{p'}} + \underbrace{\frac{1}{\rho_0 H}\left(p''_\eta \nabla \eta + p''_d \nabla d\right)}_{\text{coupling terms}} + \cdots$$

and

$$\frac{\partial \mathbf{u}'_H}{\partial t} = -\frac{1}{\rho_\circ}\nabla p'' \underbrace{- \frac{1}{\rho_0 H}\left(p''_\eta \nabla \eta + p''_d \nabla d\right)}_{\text{coupling terms}} + \cdots .$$

- **Apart from the coupling terms, $\mathbf{U}_H$ is determined by $\overline{\overline{p'}}$ and $\mathbf{u}'_H$ is determined by p''**

  → $p^i = p''$

- **No room for $\hat{p}(x,y,z)$**

Relation between $\mathscr{C}$ and $\mathscr{P}$

Relation between $\mathscr{C}$ and $\mathscr{P}$

$\mathscr{C}$ contains the energy conversion **throughout the water column**:

Relation between $\mathscr{C}$ and $\mathscr{P}$

$\mathscr{C}$ contains the energy conversion **throughout the water column**:

$$\mathscr{C} = \overline{g\rho' W} = -\overline{\frac{\partial p'}{\partial z}W} = -\overline{\frac{\partial p''}{\partial z}W} = -\overline{\frac{\partial p'' W}{\partial z}}$$

note: $\dfrac{\partial p'}{\partial z} = \dfrac{\partial p''}{\partial z}, \ \overline{p''} = 0$

$$= -(p''W)_\eta + (p''W)_{-d}$$

$$= \underbrace{-\left(p''_\eta \frac{\partial \eta}{\partial t} + \mathbf{U}_H \cdot (p''_\eta \nabla \eta)\right)}_{\mathscr{C}_{surface}} \underbrace{-\mathbf{U}_H \cdot (p''_{-d} \nabla d)}_{\mathscr{C}_{bottom} = \mathscr{P}}$$

Relation between $\mathscr{C}$ and $\mathscr{P}$

$\mathscr{C}$ contains the energy conversion **throughout the water column**:

$$\mathscr{C} = \overline{g\rho' W} = -\overline{\frac{\partial p'}{\partial z} W} = -\overline{\frac{\partial p''}{\partial z} W} = -\overline{\frac{\partial p'' W}{\partial z}}$$

note: $\frac{\partial p'}{\partial z} = \frac{\partial p''}{\partial z}, \ \overline{p''} = 0$

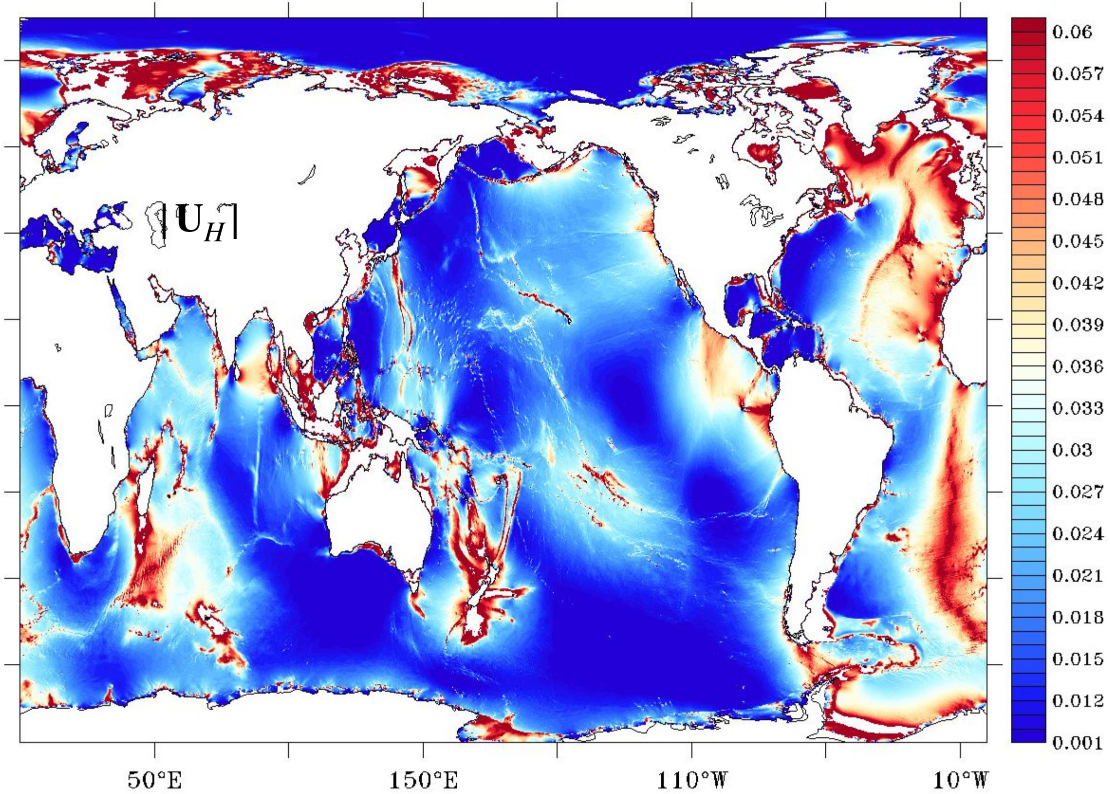
$$= -(p''W)_\eta + (p''W)_{-d}$$

$$= \underbrace{-\left(p''_\eta \frac{\partial \eta}{\partial t} + \mathbf{U}_H \cdot (p''_\eta \nabla \eta)\right)}_{\mathscr{C}_{surface}} \underbrace{-\mathbf{U}_H \cdot (p''_{-d} \nabla d)}_{\mathscr{C}_{bottom} = \mathscr{P}}$$

- **$\mathscr{C}$ includes the from drags induced by p'' at both the surface and the bottom: $\mathscr{C} = \mathscr{C}_{surface} + \mathscr{C}_{bottom}$ with $\mathscr{C}_{bottom} = \mathscr{P}$**
- **When concentrating on the IT-generation at the bottom, the effect due to surface form drag has to be excluded (as suggested by Kelly et al. 2010, but not via an additional $\hat{p}$)**

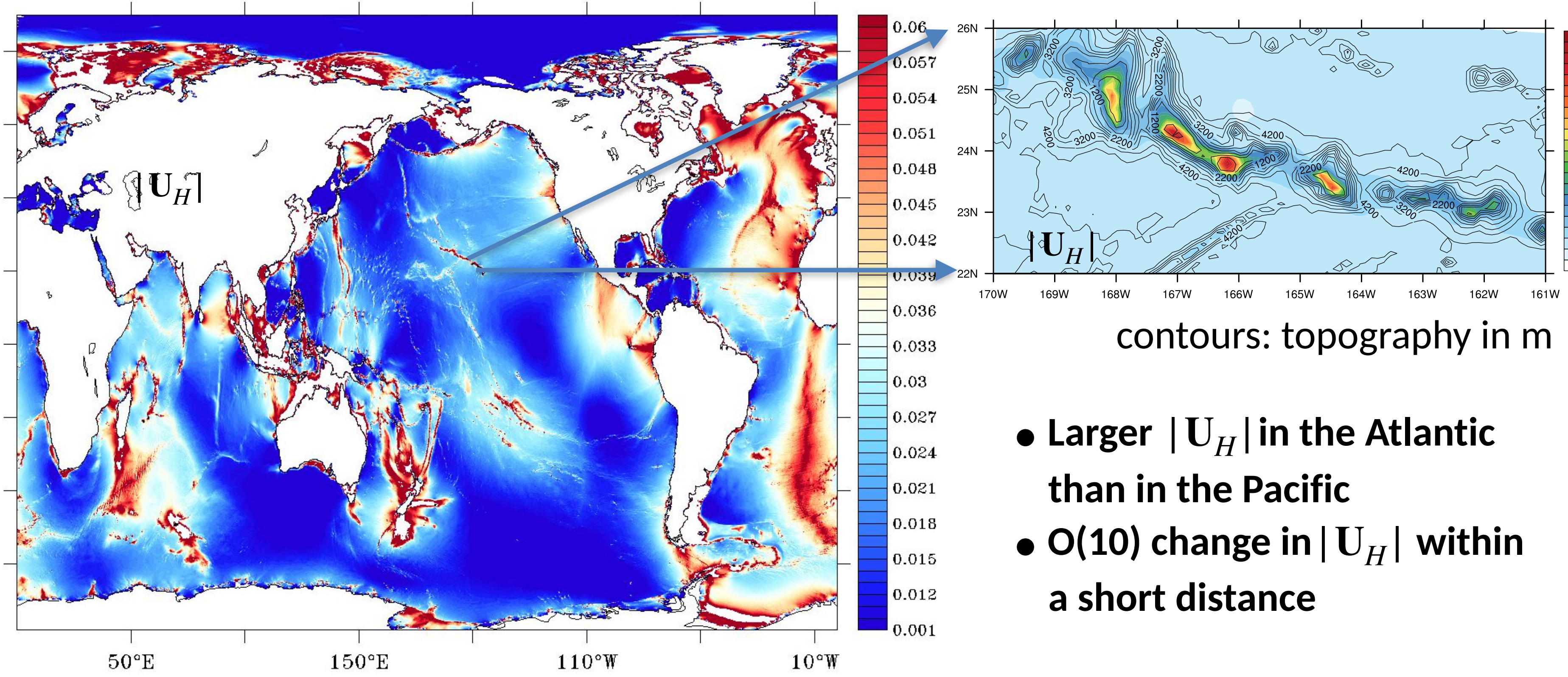
IT-generation in STORMTIDE2: horizontal variations of $|\mathbf{U}_H|$ in m/s

IT-generation in STORMTIDE2: horizontal variations of $|\mathbf{U}_H|$ in m/s



- **Larger $|\mathbf{U}_H|$ in the Atlantic than in the Pacific**

IT-generation in STORMTIDE2: horizontal variations of $|\mathbf{U}_H|$ in m/s

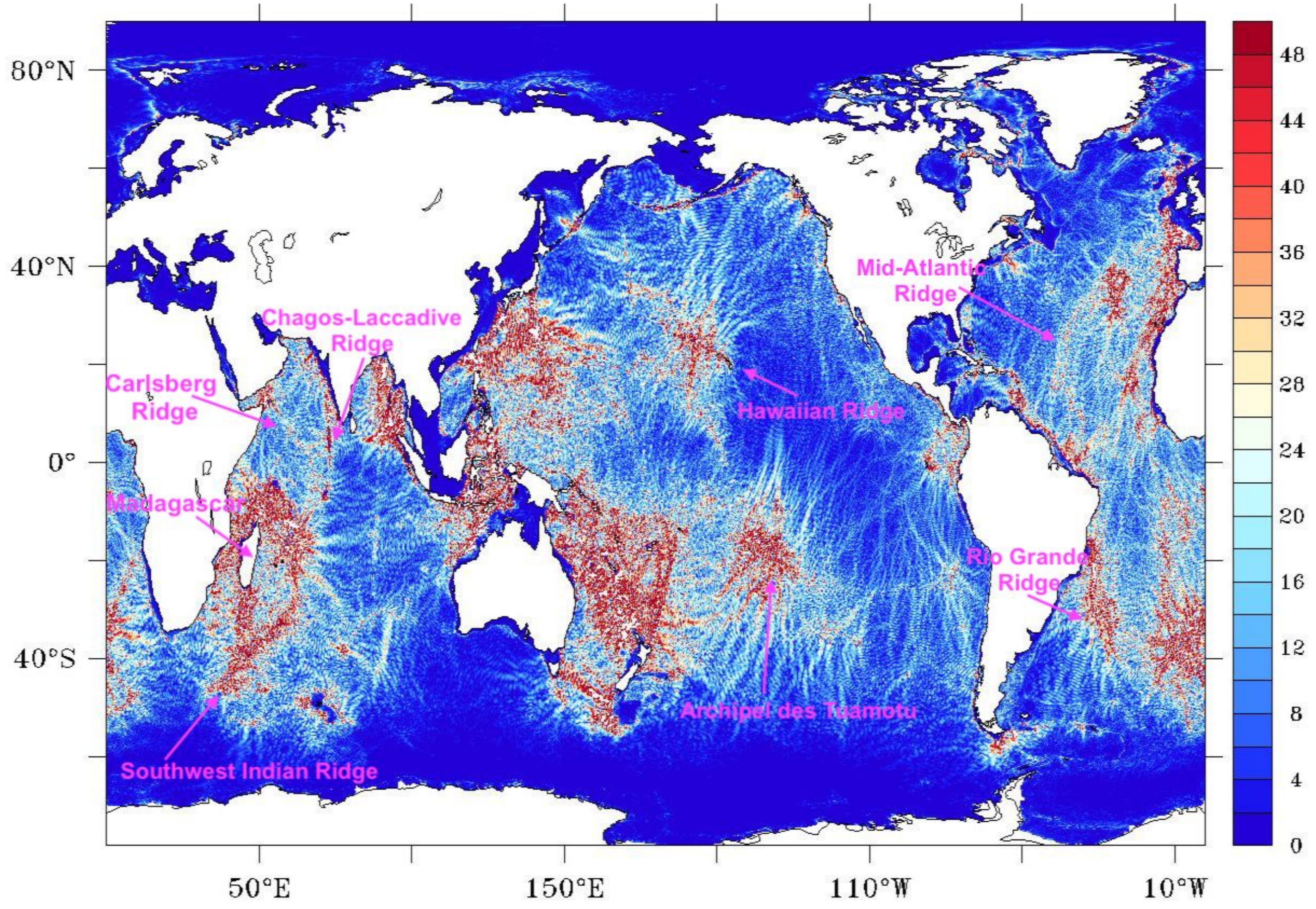


contours: topography in m

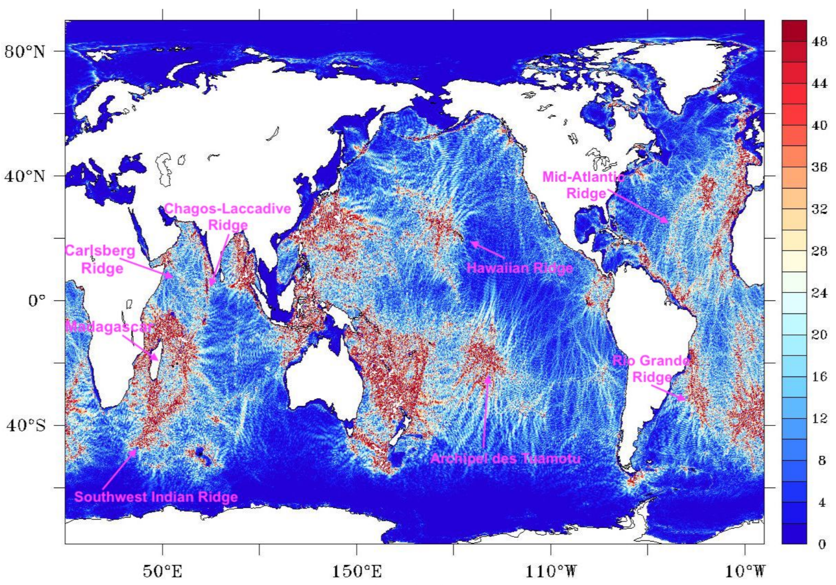
- **Larger $|\mathbf{U}_H|$ in the Atlantic than in the Pacific**
- **O(10) change in $|\mathbf{U}_H|$ within a short distance**

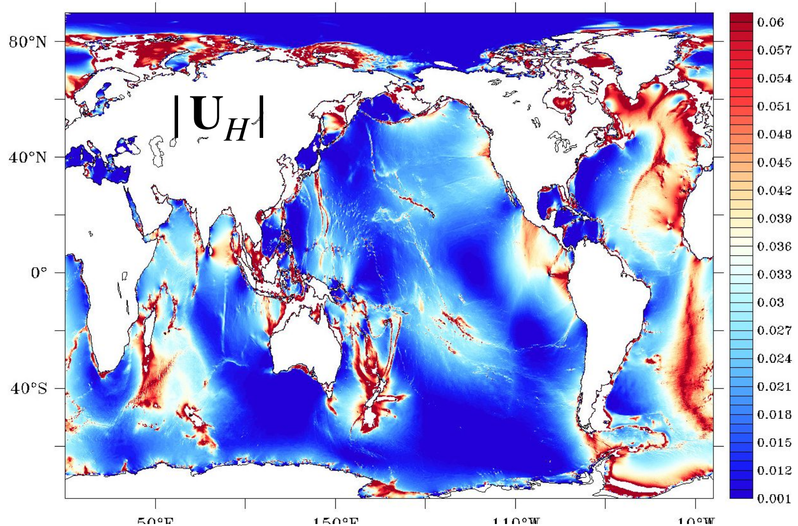
IT-generation in STORMTIDE2: p''_{-d} in N/m²

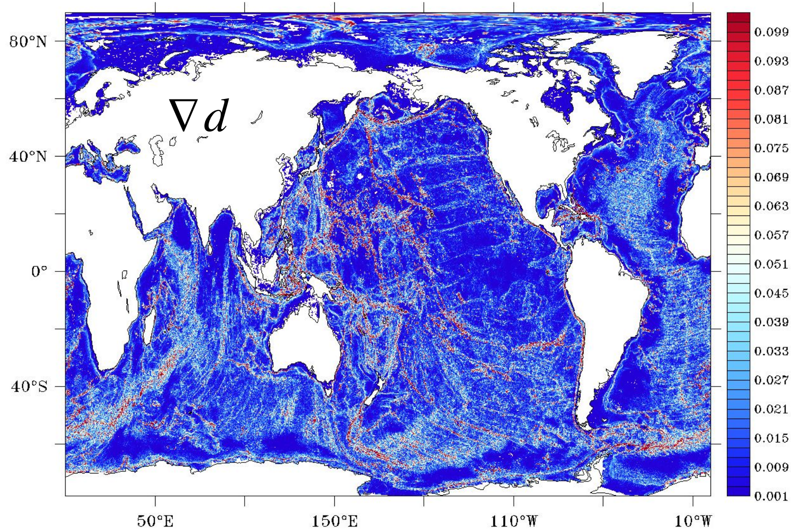
IT-generation in STORMTIDE2: p''_{-d} in N/m²



IT-generation in STORMTIDE2: p''_{-d} in N/m²







- p''_{-d} **is affected by both** $|\mathbf{U}_H|$ **and** ∇d
- **The effect of** $|\mathbf{U}_H|$ **seems to be more direct than that of** ∇d

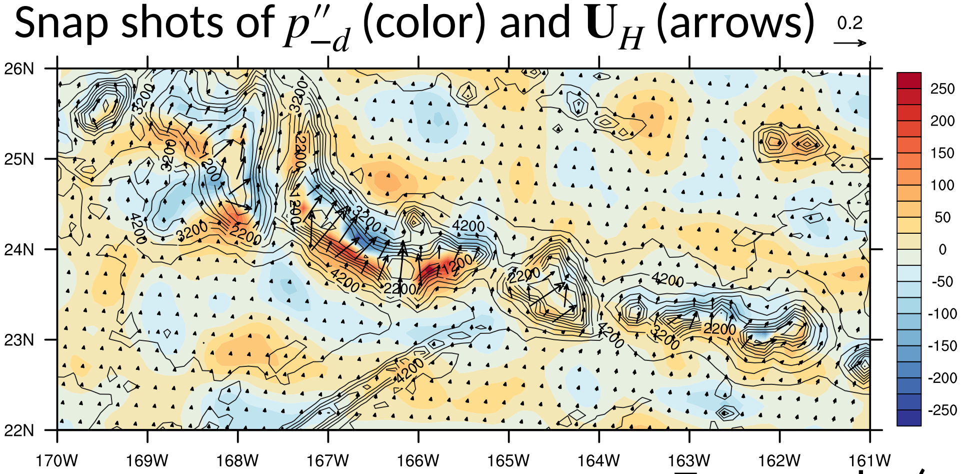
IT-generation in STORMTIDE2: phase of p''

IT generation = work done by the bottom form drag?

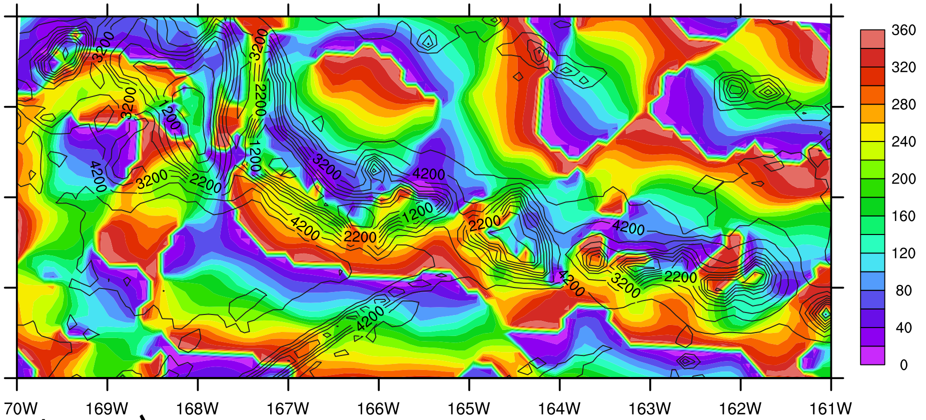
IT-generation in STORMTIDE2: phase of p''

IT generation = work done by the bottom form drag?

Snap shots of p''_{-d} (color) and $\mathbf{U}_H$ (arrows)



Phase of p''_{-d}

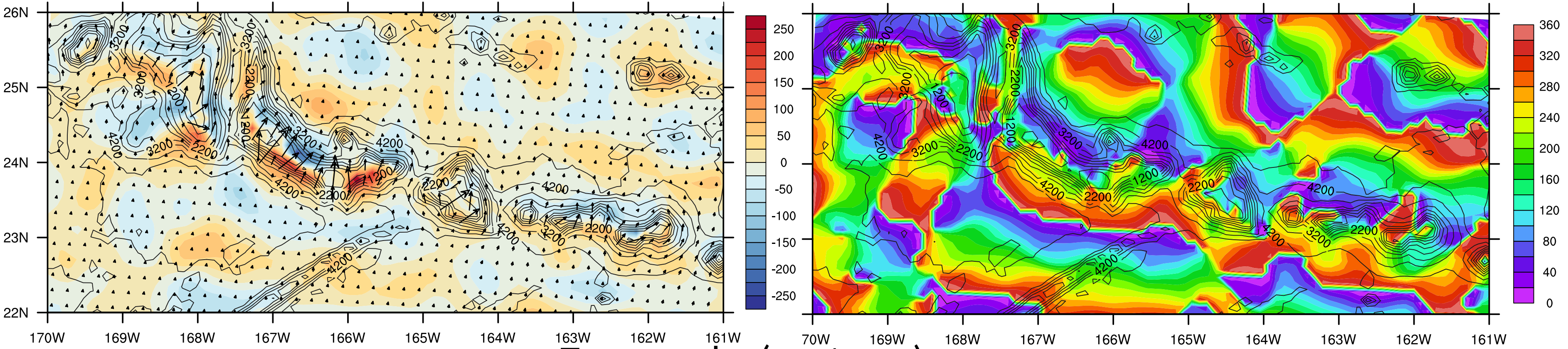


Topography (contours)

IT-generation in STORMTIDE2: phase of p''

IT generation = work done by the bottom form drag?

Snap shots of p''_{-d} (color) and $\mathbf{U}_H$ (arrows) — Phase of p''_{-d}

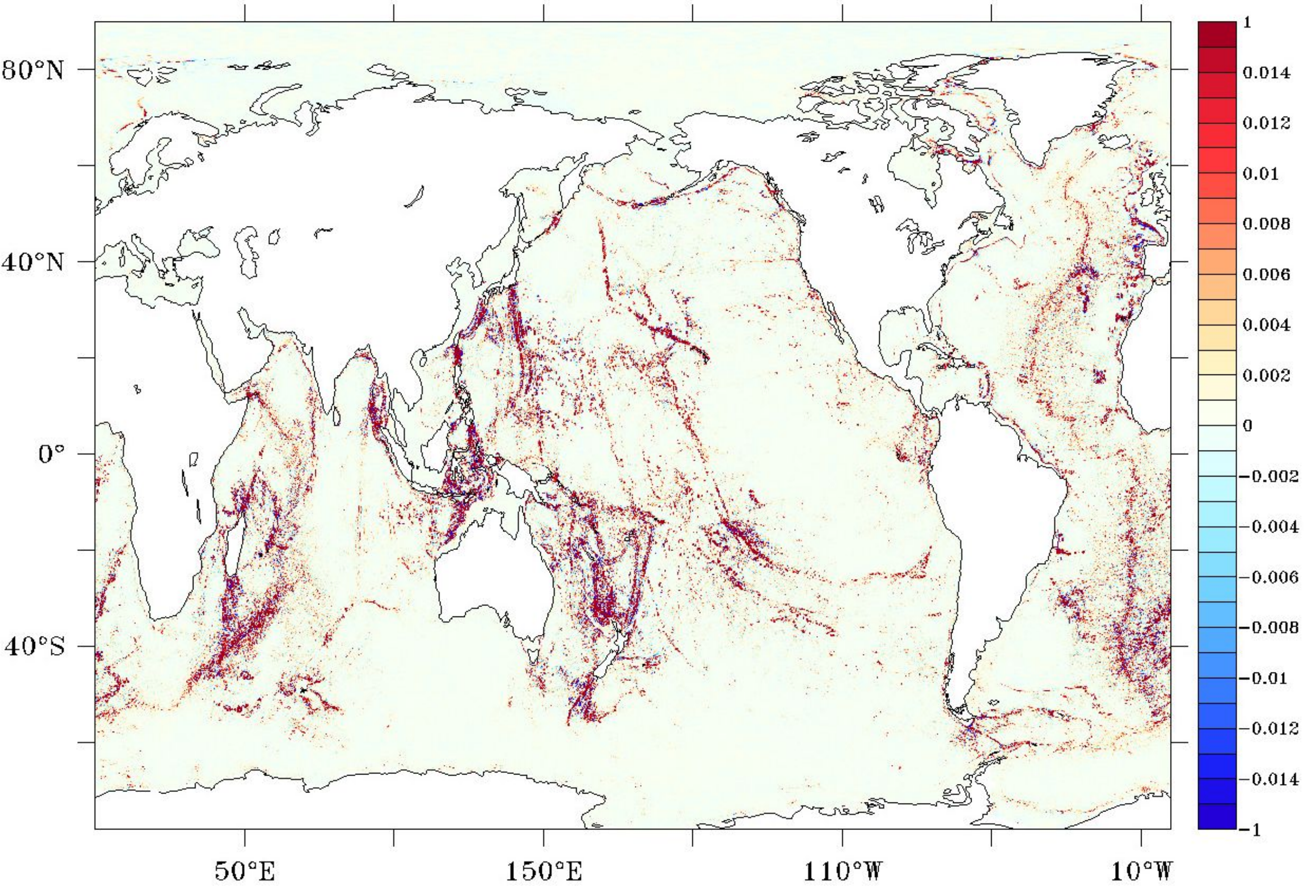


Topography (contours)

- **Bottom form drag is at work: p''_{-d} drops from the windward to the leeward side**
- **~ 180° phase shift from the windward to the leeward side**

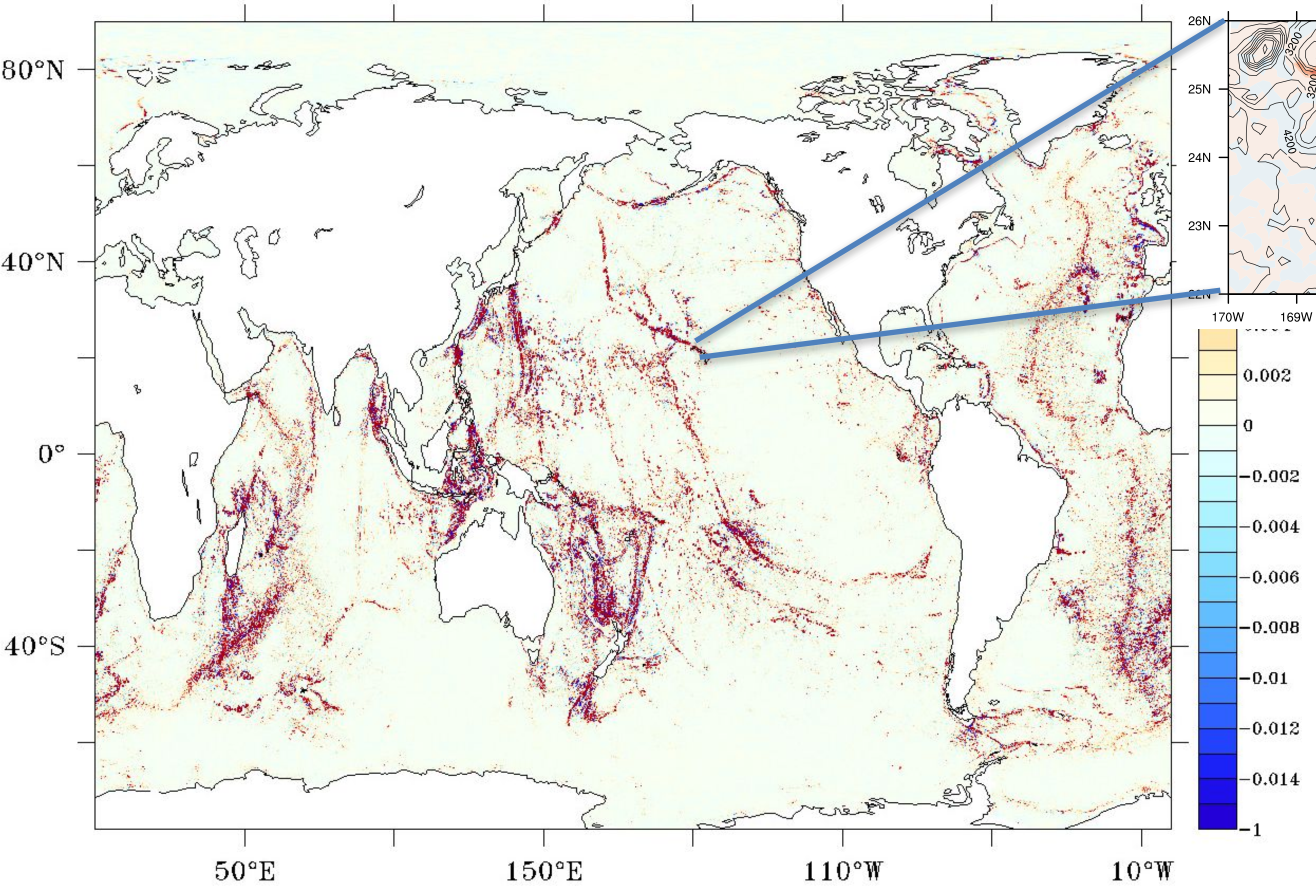
IT-generation in STORMTIDE2: $\mathscr{P}$ in W/m²

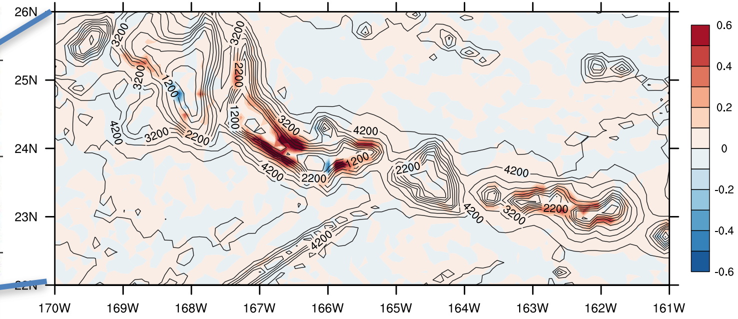
Global: 0.7 TW



IT-generation in STORMTIDE2: $\mathscr{P}$ in W/m²

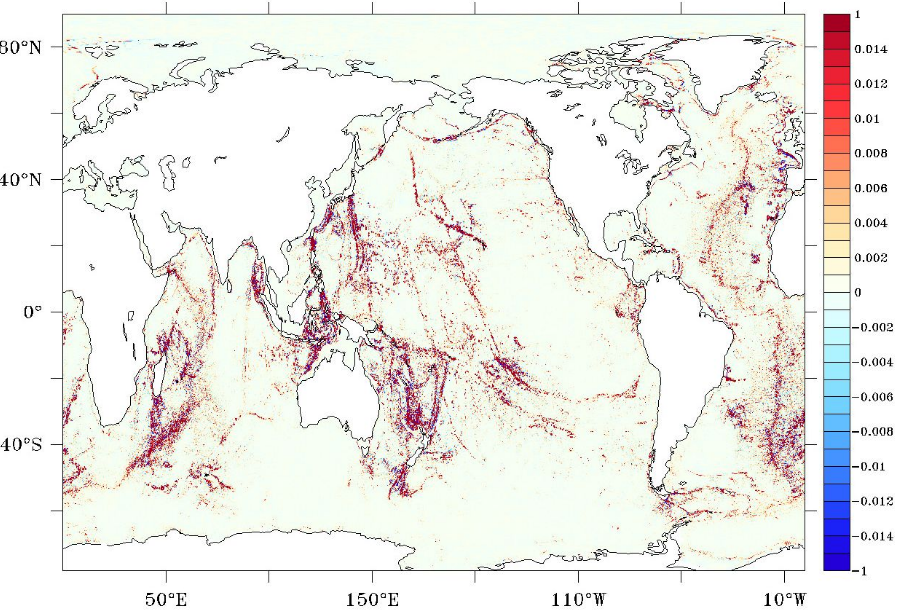
Global: 0.7 TW

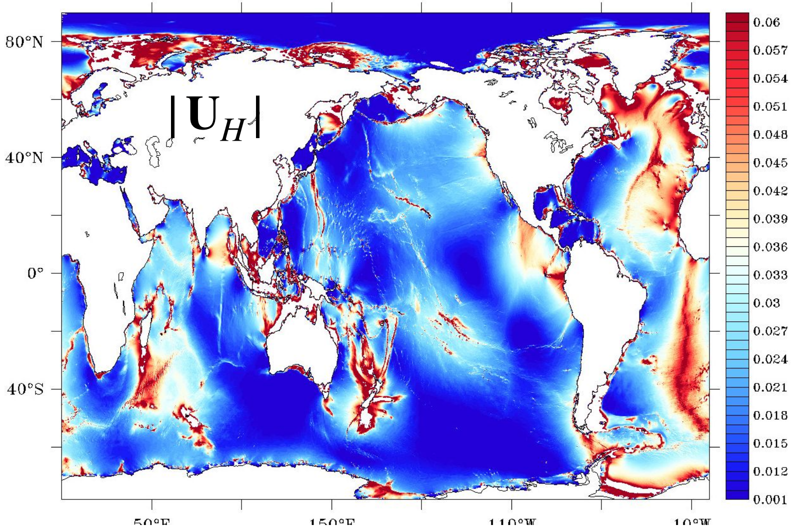


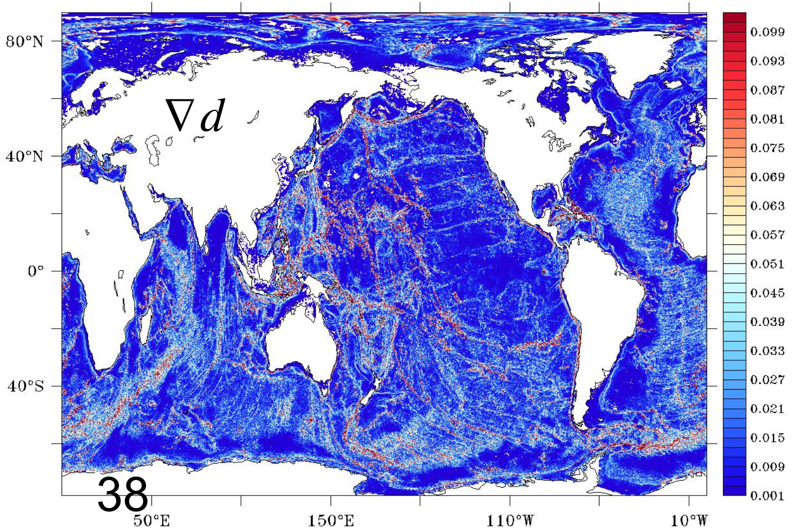


- **Extremely localized**

IT-generation in STORMTIDE2: $\mathscr{P}$ in W/m²





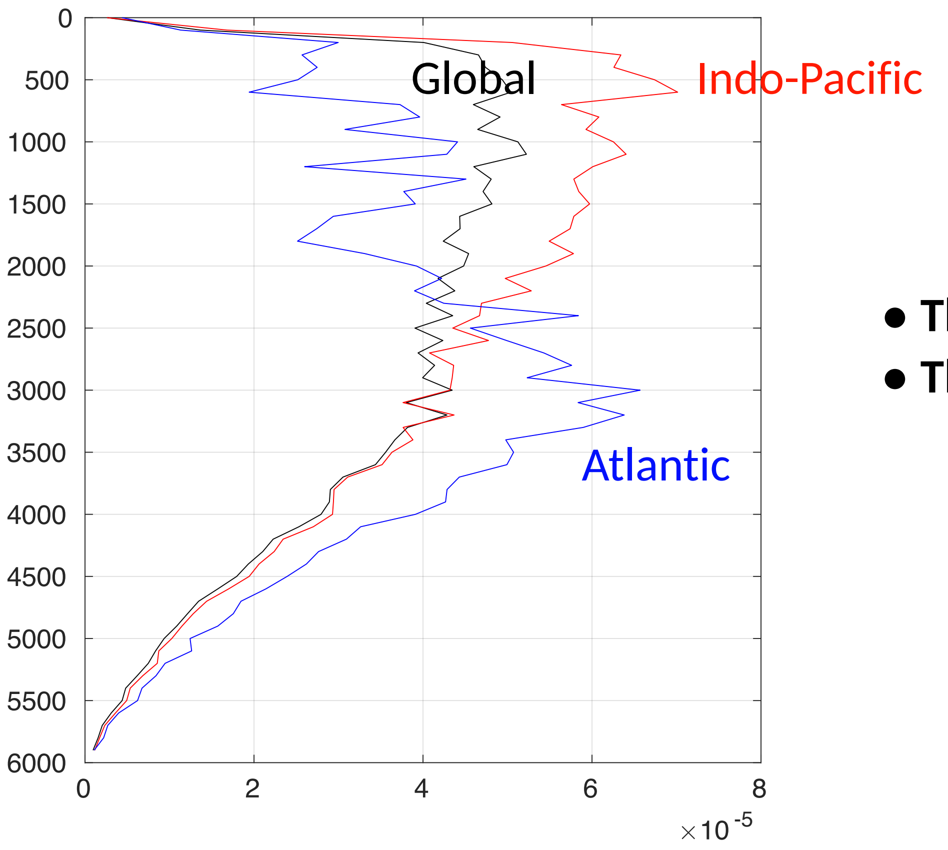


- **$\mathscr{P}$ is controlled by both $|\mathbf{U}_H|$ and ∇d**
- **The control of $|\mathbf{U}_H|$ seems to be more direct than that of ∇d**

IT-generation in STORMTIDE2: depth structure in different basins in W/m^2

(At which depth is the strongest IT-generation located?)

IT-generation in STORMTIDE2: depth structure in different basins in W/m^2
(At which depth is the strongest IT-generation located?)



- **The Indo-Pacific is heated from above**
- **The Atlantic is heated from below**

Conclusions concerning our understanding:

- Energy consideration depends on the decomposition of w
- The energy conversion $\mathscr{C}$ contains the work done by the bottom form drag $\mathscr{C}_{bottom} = \mathscr{P}$ and the work by the surface form drag $\mathscr{C}_{surface}$
- $\mathscr{C}_{surface}$ is about 1% of $\mathscr{P}$ in STORMTIDE2 (Note $p_{atm} = 0$ in MPIOM)
- $p^i = p'' = p' - \overline{p'}$

Conclusions concerning our understanding:

- Energy consideration depends on the decomposition of w
- The energy conversion $\mathscr{C}$ contains the work done by the bottom form drag $\mathscr{C}_{bottom} = \mathscr{P}$ and the work by the surface form drag $\mathscr{C}_{surface}$
- $\mathscr{C}_{surface}$ is about 1% of $\mathscr{P}$ in STORMTIDE2 (Note $p_{atm} = 0$ in MPIOM)
- $p^i = p'' = p' - \overline{p'}$

Conclusions concerning IT-generation in STORMTIDE2:

- Bottom form drag is at work
- $\mathscr{P}$ is more directly controlled by $|\mathbf{U}_H|$
- The strongest IT-generation is located at about 3000 m in the Atlantic, but at about 500 m in the Indo-Pacific

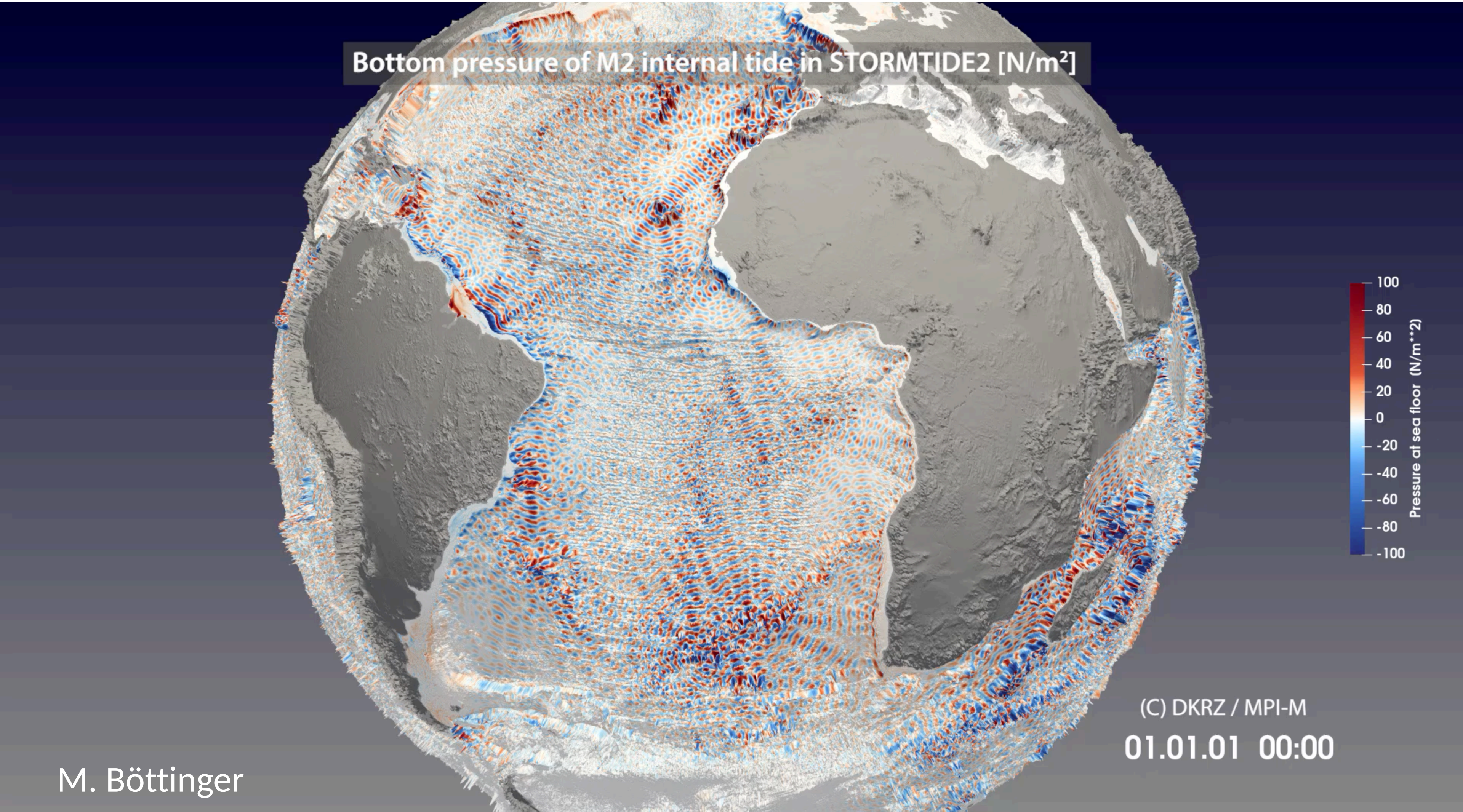
Conclusions concerning our understanding:

- Energy consideration depends on the decomposition of w
- The energy conversion $\mathscr{C}$ contains the work done by the bottom form drag $\mathscr{C}_{bottom} = \mathscr{P}$ and the work by the surface form drag $\mathscr{C}_{surface}$
- $\mathscr{C}_{surface}$ is about 1% of $\mathscr{P}$ in STORMTIDE2 (Note $p_{atm} = 0$ in MPIOM)
- $p^i = p'' = p' - \overline{p'}$

THANKS!

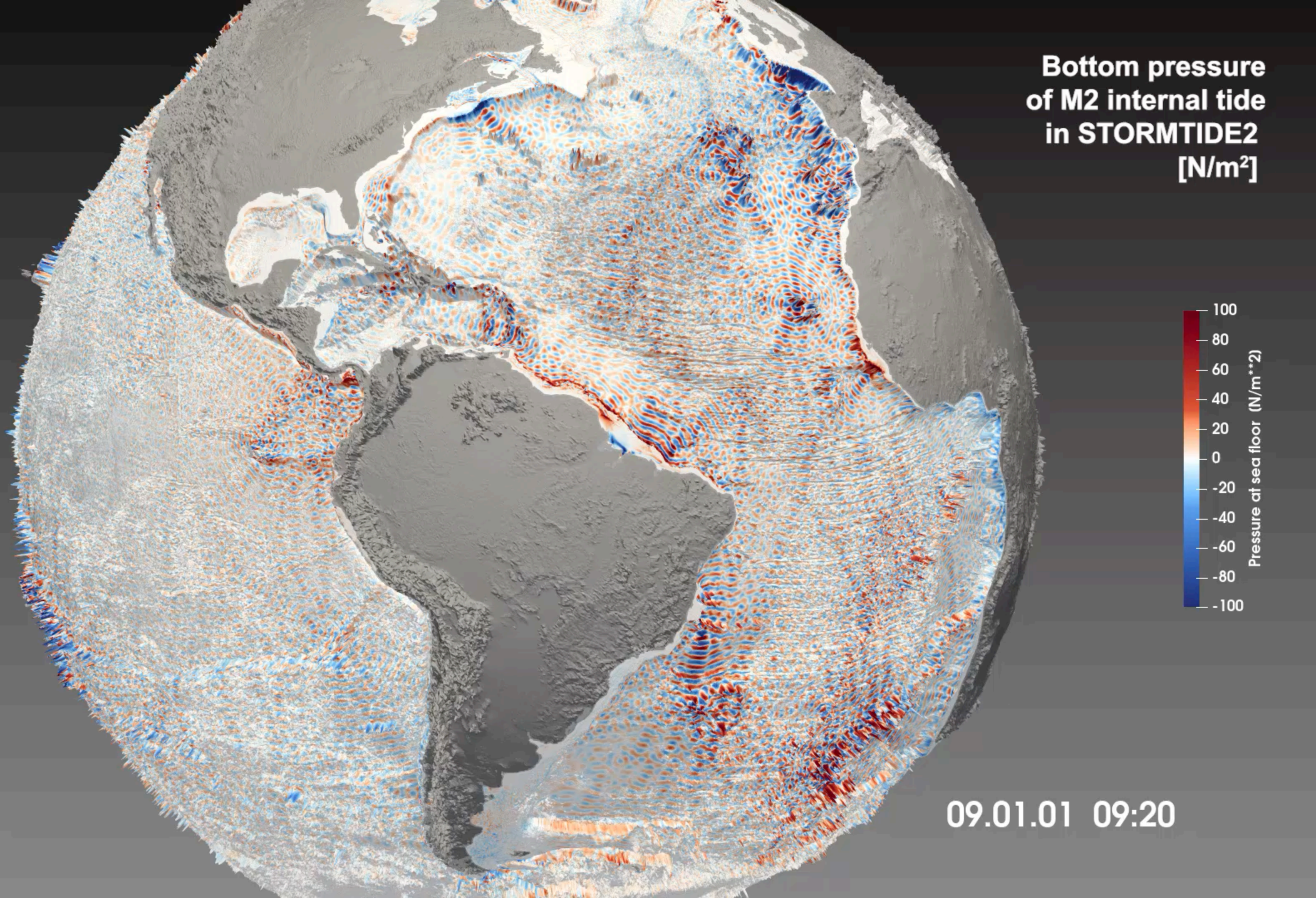
Conclusions concerning IT-generation in STORMTIDE2:

- Bottom form drag is at work
- $\mathscr{P}$ is more directly controlled by $|\mathbf{U}_H|$
- The strongest IT-generation is located at about 3000 m in the Atlantic, but at about 500 m in the Indo-Pacific



Bottom pressure of M2 internal tide in STORMTIDE2 [N/m²]
100
80
60
40
20
0
-20
-40
-60
-80
-100
Pressure at sea floor (N/m**2)
(C) DKRZ / MPI-M
01.01.01 00:00

M. Böttinger


Bottom pressure
of M2 internal tide
in STORMTIDE2
[N/m²]
100
80
60
40
20
0
-20
-40
-60
-80
-100
Pressure at sea floor (N/m**2)
09.01.01 09:20

M. Böttinger