Ocean Modelling effort at MIT:
Efficient LES model and suitable test-cases

Jean-Michel Campin, Chris Hill, John Marshall
Ali Ramadhan & Gregory Wagner,

M.I.T., Cambridge, MA, USA

Hamburg COMMODORE conference

Jan 28-31, 2020

Plan

Part I : Oceananigans, a new LES model

- Motivation
- Formulation
- Some early aplications

Part II : test cases

- Simple test case
- Full dynamical test case
- Missing oceanic tests

Need for a new LES model

- A new climate modeling effort (CLImate Modeling Alliance, https://clima.caltech.edu/) intended to improve climate models, specially, SGS parametrisations.
- One approach (among others): to use LES model results to improve and train parameterisations, e.g., OGCM diapycnal mixing closure.
- Requires efficient LES to go through multiple seasonal cycles using few-meter scale resolution.

Strategy :

- for simple domain geometry simulations (e.g., box, reentrant channel or doublelly periodic), use spectral method for the pressure solver.
- written in julia to be able to run on mixed architecture platforms, both on CPU and GPU, thus taking advantage of faster GPU hardware.

Oceananigans model formulation

(https://github.com/climate-machine/Oceananigans.jl)

- Non-hydrostatic, boussinesq primitive equations
- for now, rigid-lid but easy to add linear free-surface
- uniform horizontal resolution, simple domain geometry
- simplified (linear) or realistic oceanic EOS
- Numerical discretization, similar to MITgcm: C-grid, finaite volume, implicit non-hydrostatic pressure solver, Adams-bathforth time stepping for explicit tendencies.
- Direct 3-D pressure solver
- Currently, only 2^{nd} order centered advection scheme;
  5^{th} Order WENO scheme available soon

Direct pressure solver

kinematic pressure:

$\phi = P/\rho_c = \phi_{\rm nh} + \phi'_{\rm hyd} - gz \quad$ with : $\quad \phi'_{\rm hyd} = \int_z^0 (\rho(\theta, S, z) - \rho_c)/\rho_c \, g \, dz$

$$\nabla \cdot \mathrm{v}^{n+1} = 0 \tag{1}$$

$$\mathrm{v}^{n+1} + \Delta t \nabla \phi_{\rm nh}^{n+1} = \mathrm{v}^n + \Delta t \mathrm{G_v} \tag{2}$$

$$\text{with :} \quad \mathrm{G_v} = -\nabla_h \phi'_{\rm hyd} - (\mathrm{v} \cdot \nabla)\,\mathrm{v} - \mathrm{f} \times \mathrm{v} + \nabla \cdot \tau + \mathrm{F_v}$$

$$(1)\ \&(2) \Rightarrow \quad \nabla \cdot (\nabla \phi_{\rm nh}^{n+1}) = \nabla \cdot (\mathrm{v}^n + \Delta t \mathrm{G_v})/\Delta t = S \tag{3}$$

Case with uniform resolution in each direction:

Numerical solution (discretized in space) $\phi_{i,j,k} = \phi(i\Delta_x, j\Delta y, k\Delta z)$ is expressed exactly in tem of Discrete Fourrier series:

$$\phi_{i,j,k} = \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \sum_{p=1}^{N_z} \widehat{\phi}^{m,n,p} \ e^{-ik_x^m i \Delta x} e^{-ik_y^n j \Delta y} e^{-ik_z^p k \Delta z}$$

with: $k_x^m = 2\pi \frac{m}{N_x \Delta x}$, $k_y^n = 2\pi \frac{n}{N_y \Delta y}$, $k_z^p = 2\pi \frac{p}{N_z \Delta z}$

In Fourrier space, the discrete laplace operator is simply:

$\widehat{\nabla^2 \phi} = -(k_x^2 + k_y^2 + k_z^2)\widehat{\phi}$ so that equation (3) provides directly: $\widehat{\phi} = \frac{-1}{k_x^2 + k_y^2 + k_z^2} \widehat{S}$

Case with uniform resolution in horizontal directions only:

(currently, in validation stage)

Numerical solution (discretized in space) $\phi_{i,j,k}$ is expressed exactly in tem of Discrete Fourrier series in horizontal directions:

$$\phi_{i,j,k} = \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \widehat{\phi}_k^{m,n} \ e^{-ik_x^m i\Delta x} e^{-ik_y^n j\Delta y}$$

so that the discretized equation (3) gives:

$$\sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \left[-(k_x^2 + k_y^2)\widehat{\phi}_k^{m,n} + \partial_z^2 \widehat{\phi}_k^{m,n} - \widehat{S}_k^{m,n} \right] e^{-ik_x^m i\Delta x} e^{-ik_y^n j\Delta y} = 0$$

And using a simple discretisation of
$\partial_z^2 \phi \quad \leftrightarrow \quad [(\phi_{k+1} - \phi_k)/\Delta_z^{k+1/2} - (\phi_k - \phi_{k-1})/\Delta_z^{k-1/2}]/\Delta_z^k$
gives for each horizontal mode m, n:

$$\frac{\widehat{\phi}_{k+1}^{m,n}}{\Delta_z^{k+1/2}} - \left[\frac{1}{\Delta_z^{k+1/2}} + \frac{1}{\Delta_z^{k-1/2}} + \Delta_z^k (k_x^2 + k_y^2) \right] \widehat{\phi}_k^{m,n} + \frac{\widehat{\phi}_{k-1}^{m,n}}{\Delta_z^{k-1/2}} = \Delta_z^k \, \widehat{S}_k^{m,n}$$

This tridiagonal system of dimention N_z is easily solved by LU decomposition.

SGS closures

Various LES SGS closures:

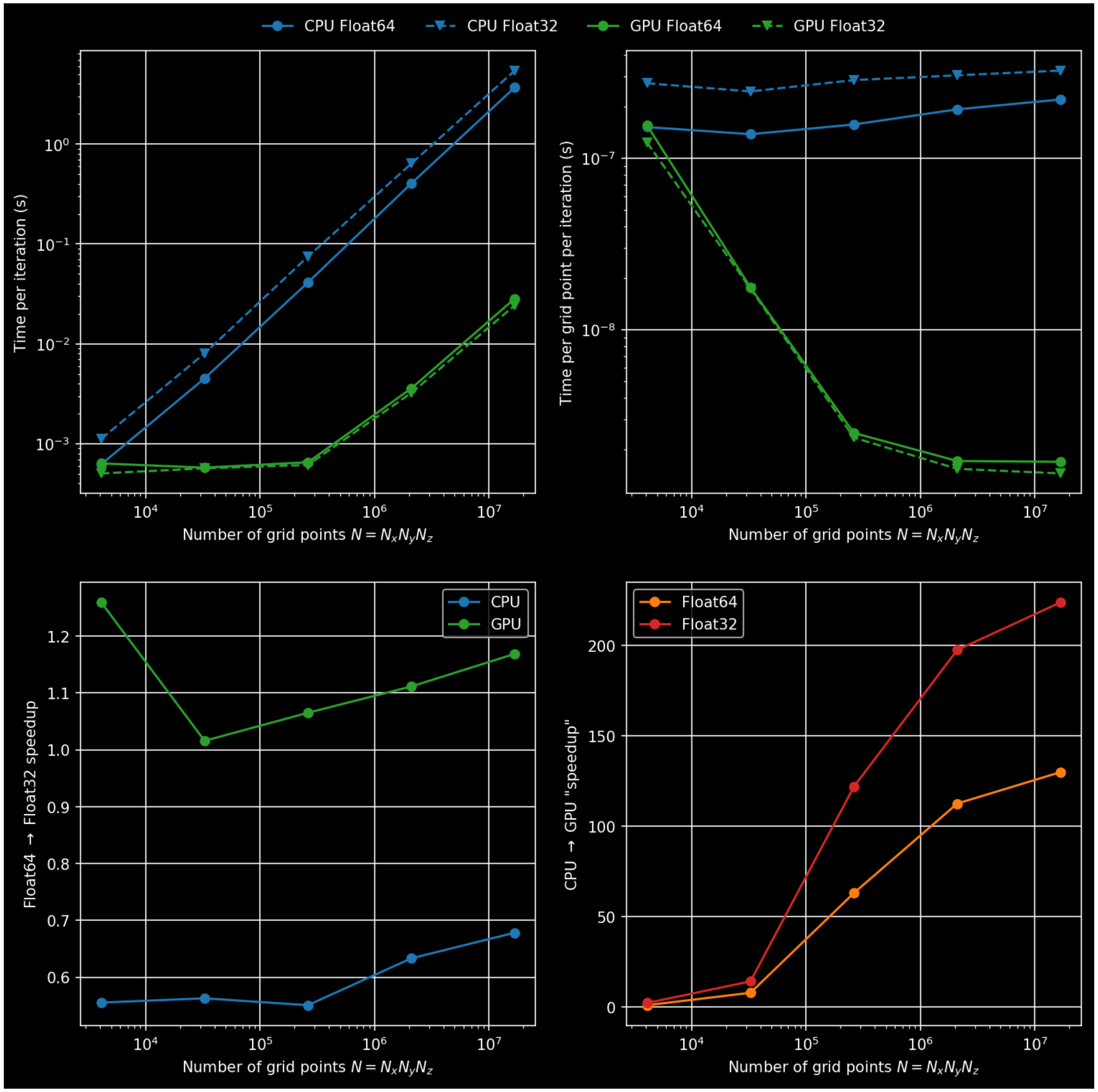
- Constant horizontal and vertical viscosity / diffusivity.
- Smagorinsky viscosity with prescribed Prandtl number for diffusivity
- Anisotropic minimum dissipation (AMD) model (Verstappen, Comput. Fluids, 2018), following imlementation of Vreugdenhil & Taylor (Phys. Fluids, 2018).

Software and performance

written in Julia

- easy to install, easy to run.
- compilation done at beginning of the run (but option to pre compile).
- great support from the Julia lab (next door) for solving performance issues.
- currently, supports CUDA (Nvidia) GPUs ; working on extension to AMD.
- currently, Oceananigans more focus on GPU optimisation (but run on both).
- MPI communication available soon in Oceananigans (open PR, under testing).
- Journal of Open Source Software (JOSS) submission under review.

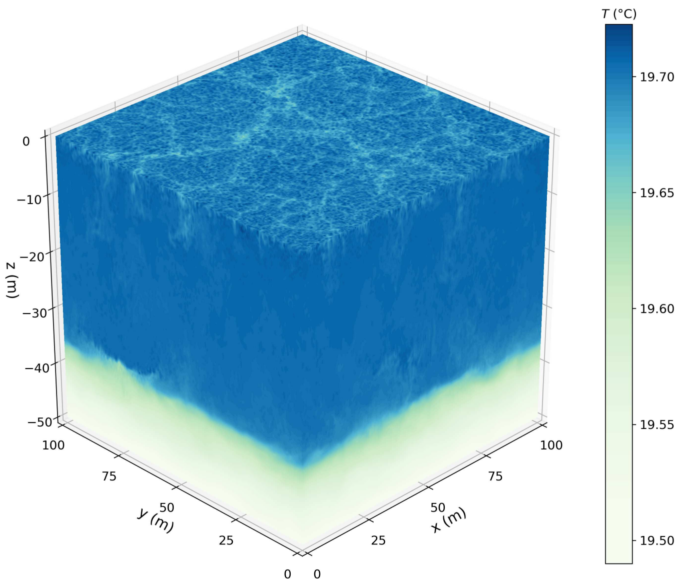
Simple test performance

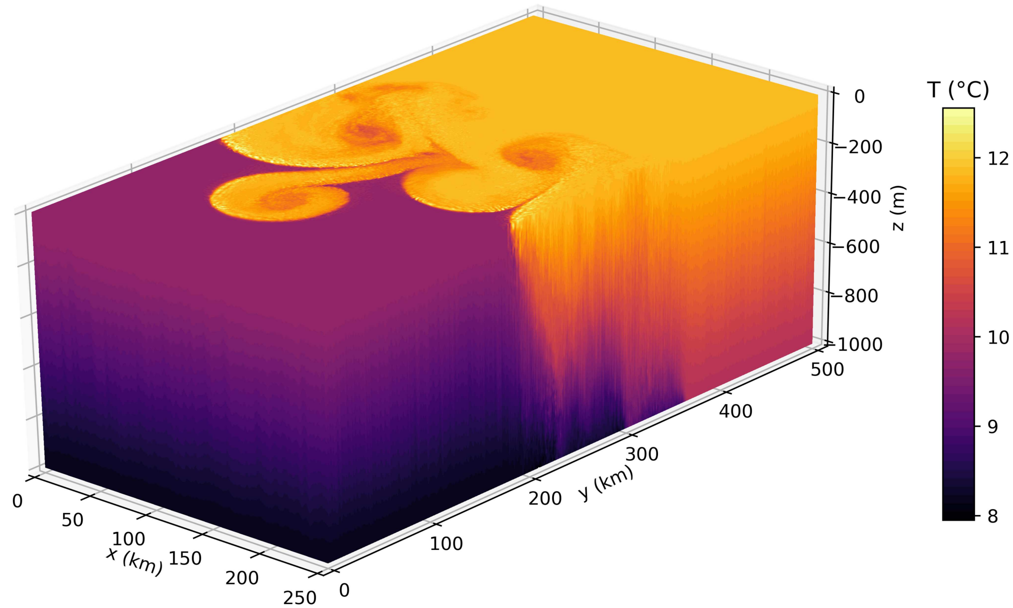


- TL: time per time-step
- TR: time per grid point per time-step

- BL: float64 / float32 speedup
- BR: GPU / CPU speedup

Early applications



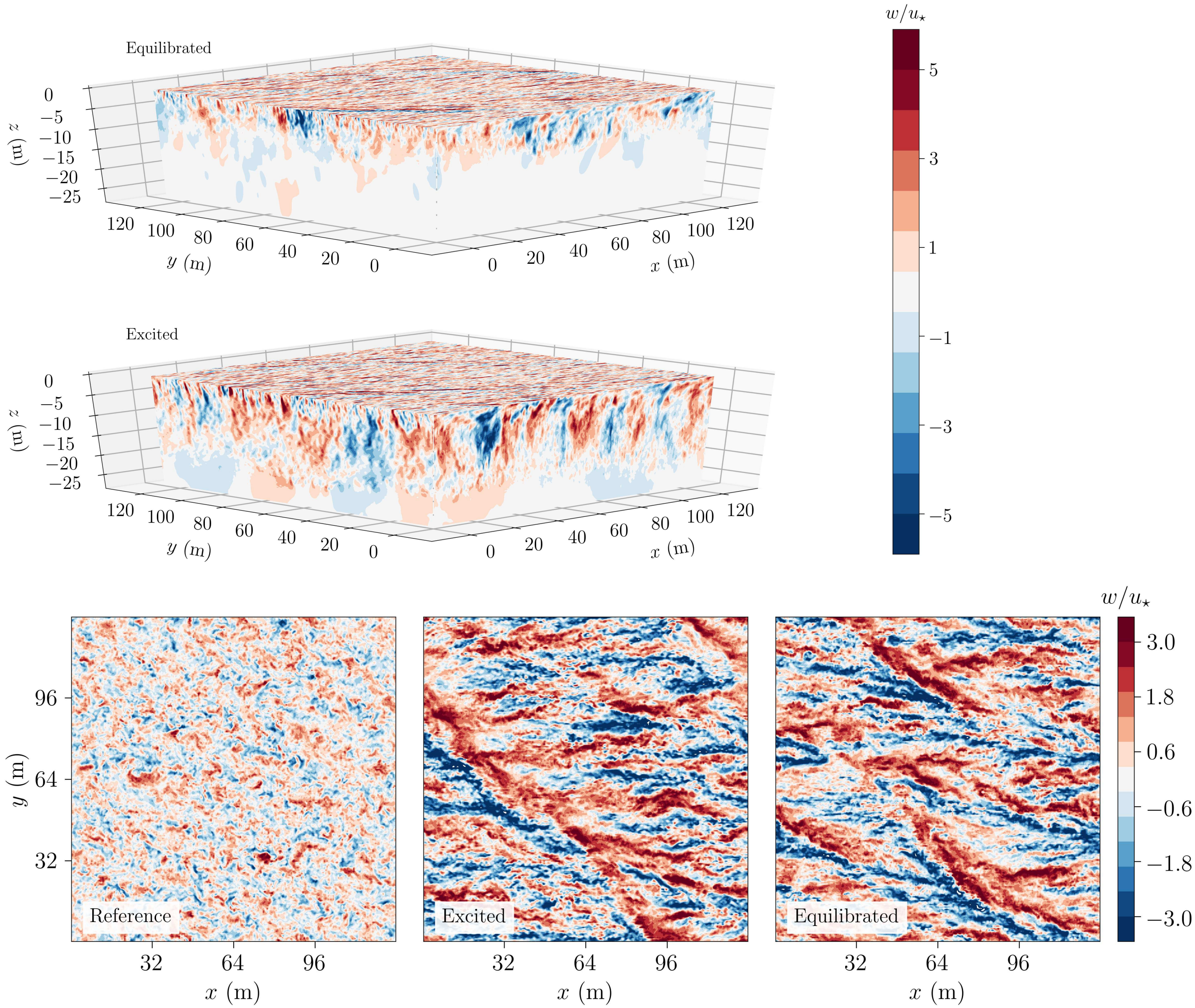


- Uncertainty quantification of some mixing scheme parameters
  → Andre Nogueira, Raffaele Ferrari
  Mixed layer deepening under uniform surface wind and cooling, in doublely periodic water column.

- Effect of meso-scale and submeso-scale eddies on surface mixing

Early applications (2)

- Effect of wind & surface waves (stoke-drift) on upper ocean mixing
  → Greg Wagner, Raffaele Ferrari



Test cases

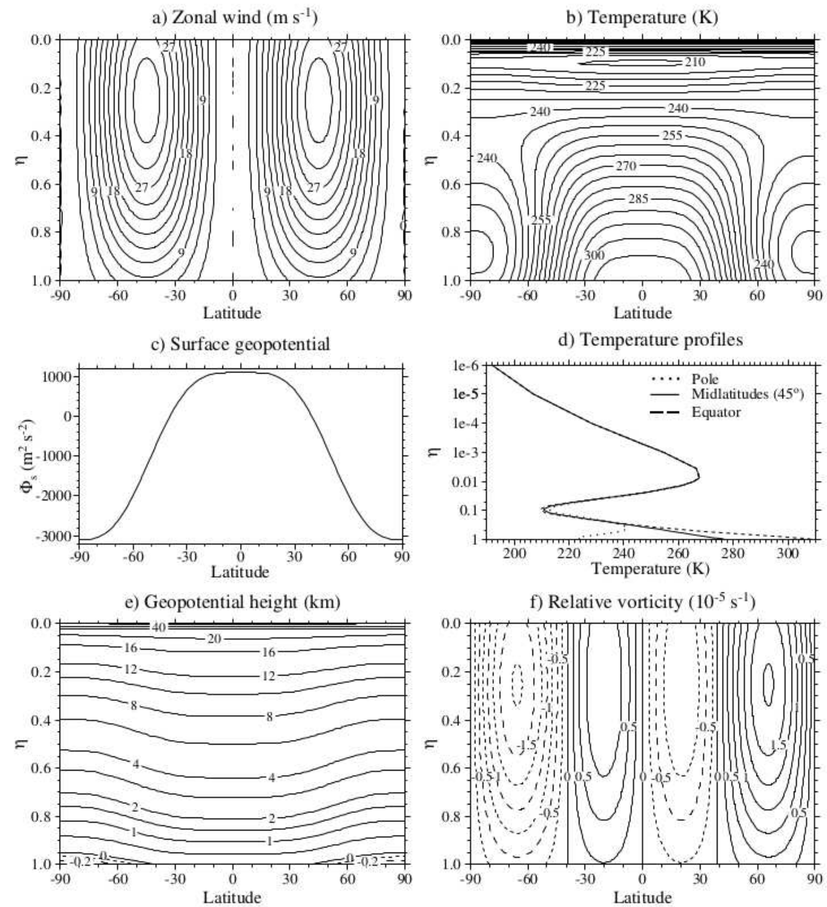
Simple tests:

- Basic operator discretization: Green-Taylor vortex.
  $\rightarrow$ analytical solution, viscous decay
- Non-Hydrostatic free-surface: short-surface wave (2-D x-z, non rotating)
- Large scale geostrophic balance (linear dynamics): Stommel gyre, Munk gyre
- Spherical geometry: solid-body rotation, tracer advection
- Many features: checking expected symetry of the solution
- for LES closure: stratified Couette flow

Full dynamics:

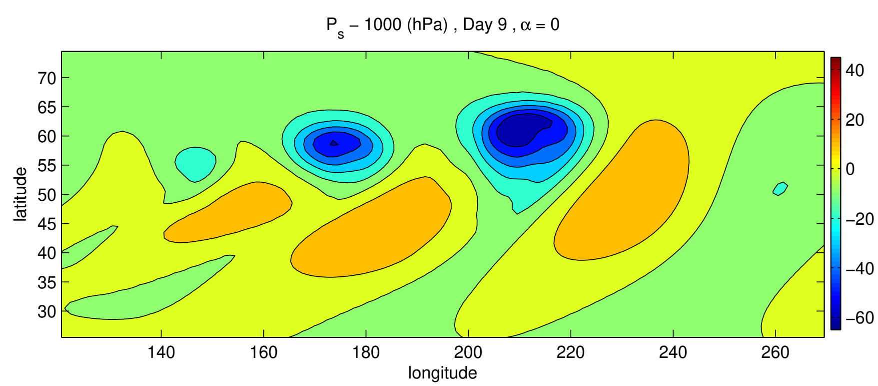
- Planetary scale atmospheric dynamics.
  Jablonowski & Williamson, QJRMS 2006 ; Lauritzen *etal*, JAMES 2010
  tests balance flow, baroclinic instability development (Rossby wave) and spherical geometry (rotating cases).
- Other atmospheric test cases proposed for DCMIP Summer School 2012 at NCAR, e.g., with idealized montains or some non-hydrostatic tests

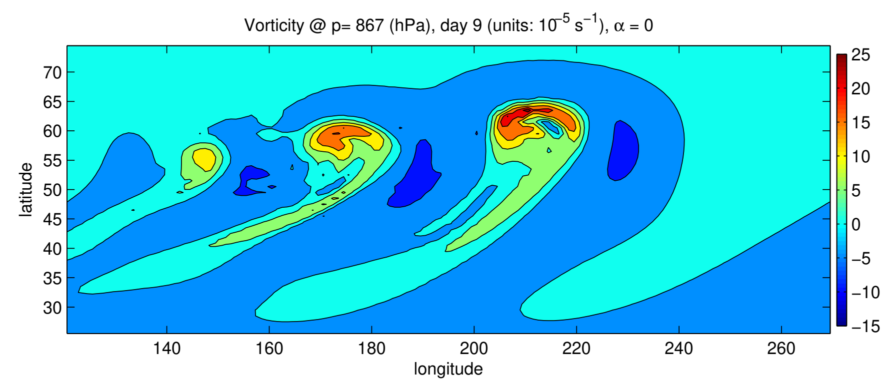
A baroclinic instability test case for atmospheric model dynamical cores



Jablonowski & Williamson, 2006

- zonally uniform, balanced initial conditions
- add depth independent perturbation in zonal velocity (centered on 40.N, 20.E)





Test cases (cont)

Missing oceanic test case (wish list):

- Do we have an equivalent good baroclinic instability test case for oceanic type conditions ?
- A good test for flow - bathymetry interactions ?
- A good test (with rotation) for wetting and drying ?

No analytial solution would be available for full dynamics test case, only model comparison (including higher resolution solution).

Conclusions

- Oceananigans, a new efficient LES model, is still in early stage development but looks promising, already usable for some scientific applications.
- Simple test cases with analytical solution or well established answer are really usefull when developing new features.
- Need inter-model comparisom to strengthen more complex/comprehensive test cases with full dynamics involved ; these might be most usuful ones.