Discrete baroclinic and symmetric instabilities on the B and C grids

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Mesoscale eddies in eddy-permitting models have too-little energy.

Viscosity is probably partly to blame.

Maybe baroclinic instability is not efficient at eddy-permitting resolution?

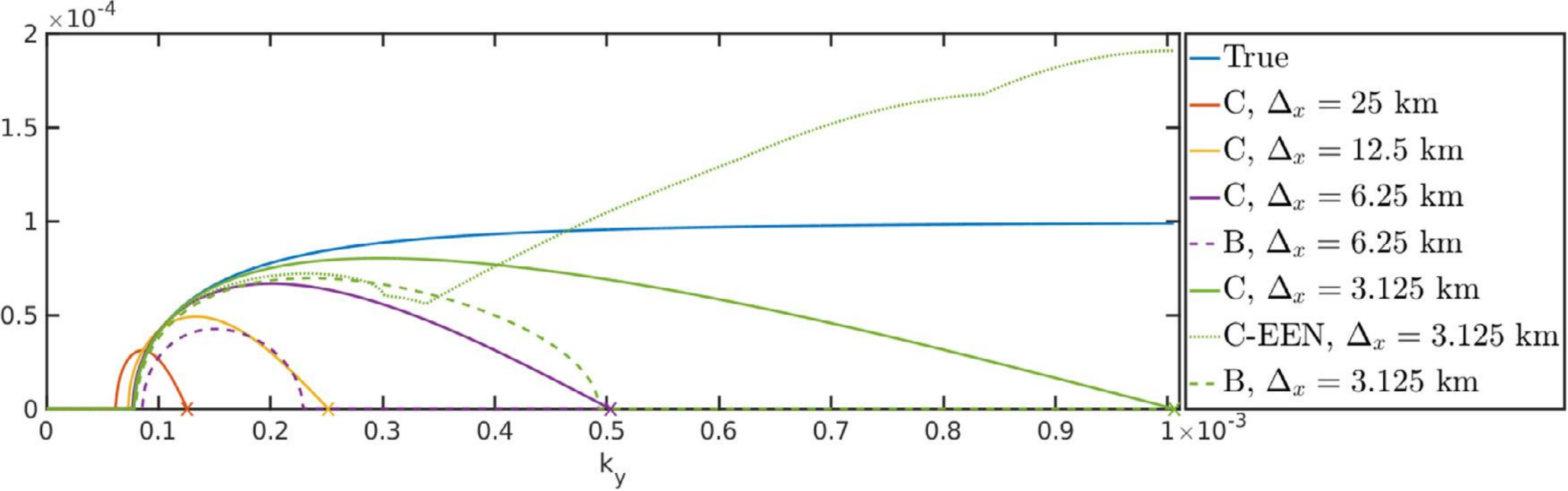
What are the growth rates of linear baroclinic instability in the **discrete** hydrostatic Eady problem?

Used large Nz to focus on effect of horizontal discretization.

Discretizations (2^{nd} order except where noted):

- B Grid, Energy Conserving (POP)
- C Grid, Energy Conserving (MOM6)
- C Grid, Energy & Enstrophy Conserving (NEMO)

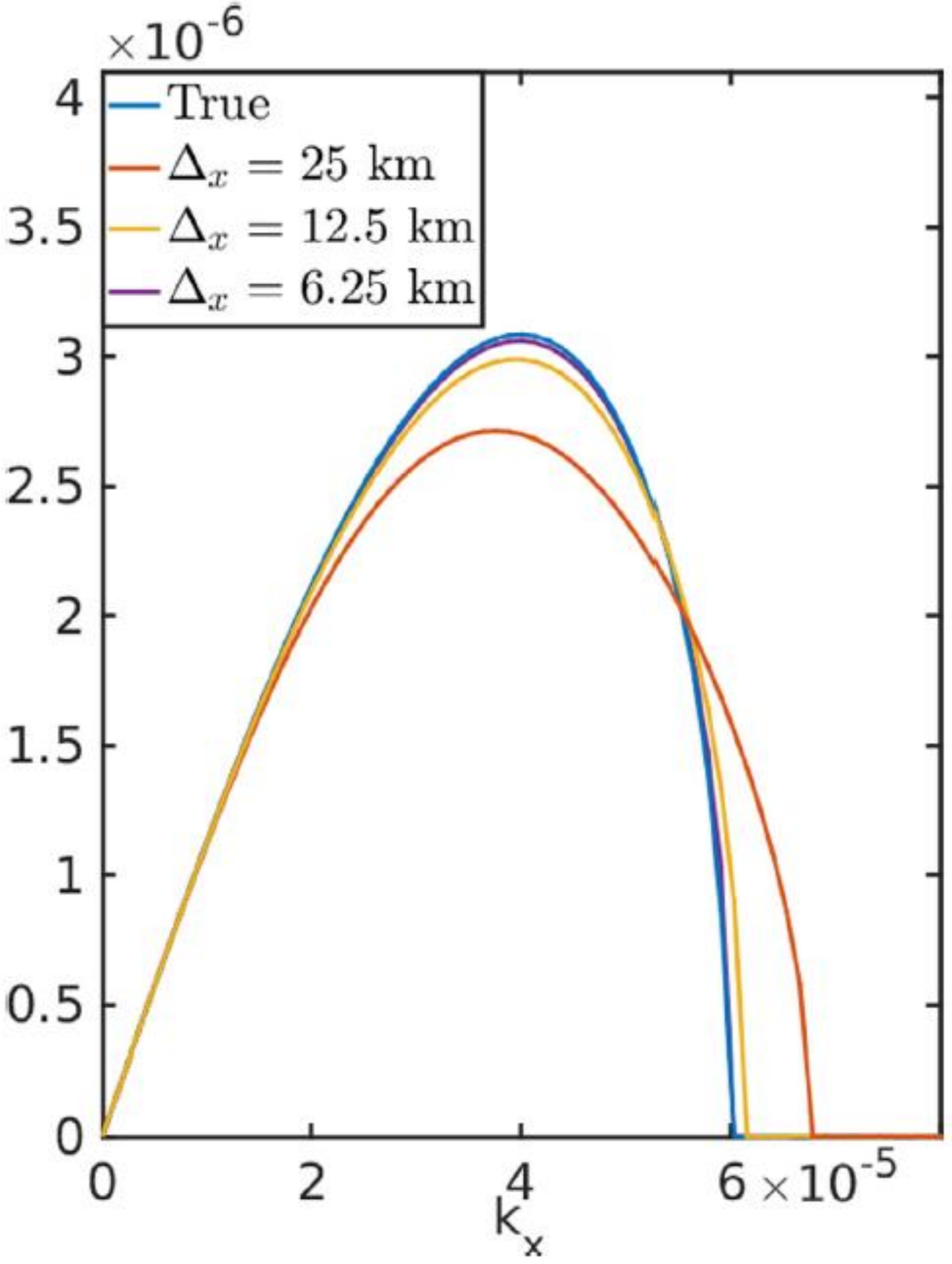
(Matlab code to generate matrices is in suppl. mat. of BBG18)



Symmetric Instability: Ri=0.5, Ld = 40 km

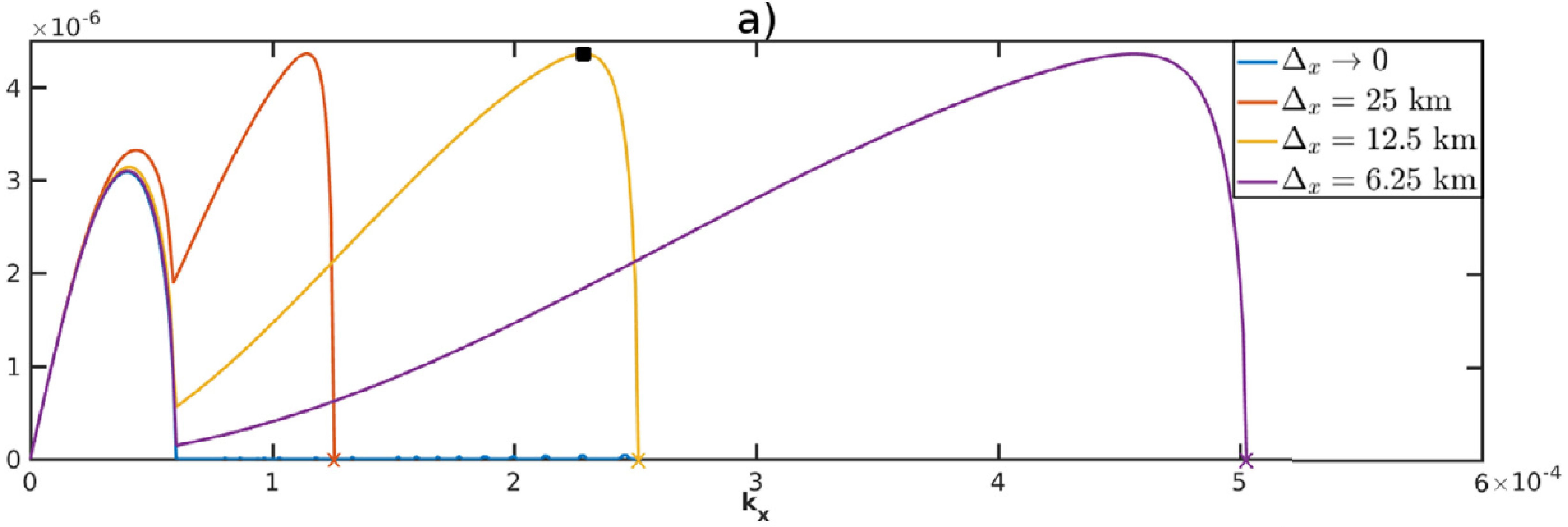
(Color=grid size, solid=C, dash=B)

- C Grid is better than B grid at fixed resolution (compare purple)
- C Grid, energy-only and energy/enstrophy are both about 100% wrong at grid scale.



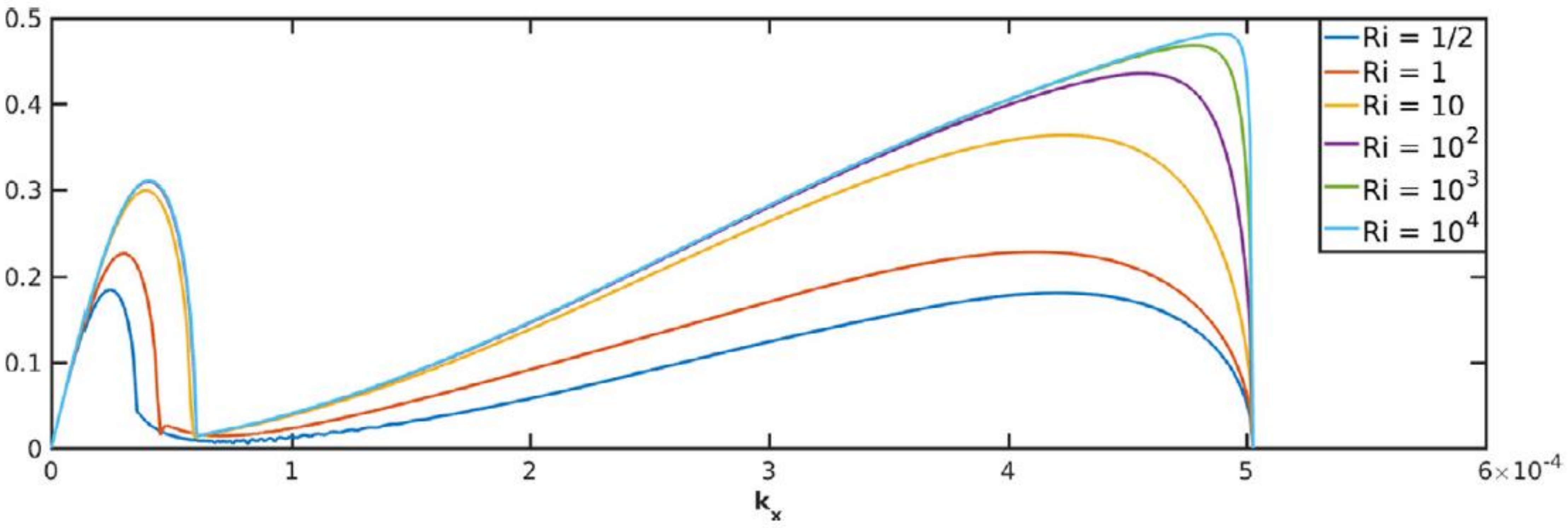
B Grid, Baroclinic:
Growth rates are indeed too small at eddy-permitting resolution.

(Ri = 100, Ld = 40 km)

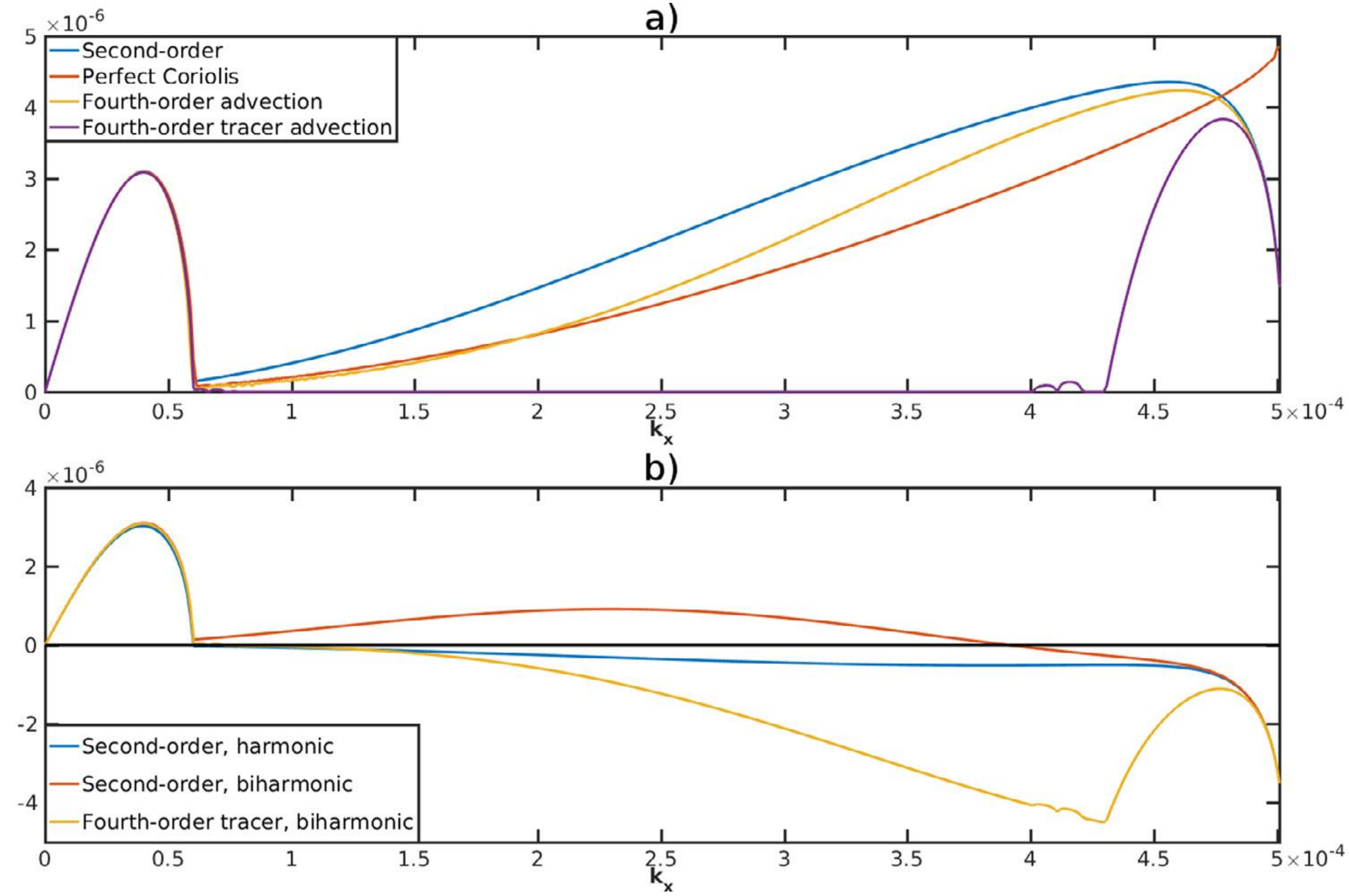


C Grid, Baroclinic: Growth rates are **too large** *and* there's a spurious instability peaked at the grid scale that is faster than the main instability!

The two C Grid discretizations are same on the baroclinic axis



C Grid, Baroclinic: Spurious growth rate seems to approach a limit as Ri->Infinity. Seems to also stay fixed as grid is refined.



C Grid, Baroclinic: 4th order tracer advection plus biharmonic viscosity seems to help.

Are the eigenvalues of the linear stability problem relevant for ocean dynamics?

The mesoscale eddies we’re trying to understand are very much *not* linear.

How do eddies extract energy from the large scales in *nonlinear* baroclinic instability, and how is this impacted by discretization?

The following is a generic eddy equation based on time averaging

$$\partial_t \boldsymbol{u}' = \mathscr{L}\boldsymbol{u}' + \mathscr{B}(\bar{\boldsymbol{u}}, \boldsymbol{u}') + \mathscr{B}(\boldsymbol{u}', \bar{\boldsymbol{u}}) + \mathscr{B}(\boldsymbol{u}', \boldsymbol{u}')' = \overline{\mathscr{L}}\boldsymbol{u}' + \mathscr{B}(\boldsymbol{u}', \boldsymbol{u}')'.$$

The eddy energy equation is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}E &= \frac{1}{2}\langle \boldsymbol{u}', \partial_t \boldsymbol{u}' \rangle + \frac{1}{2}\langle \partial_t \boldsymbol{u}', \boldsymbol{u}' \rangle = \frac{1}{2}\langle \boldsymbol{u}', \overline{\mathscr{L}}\boldsymbol{u}' \rangle + \frac{1}{2}\langle \overline{\mathscr{L}}\boldsymbol{u}', \boldsymbol{u}' \rangle \\ &= \left\langle \boldsymbol{u}', \frac{\overline{\mathscr{L}}^{\dagger} + \overline{\mathscr{L}}}{2}\boldsymbol{u}' \right\rangle,\end{aligned}$$

Eddy energy growth, e.g. through nonlinear baroclinic instability, is mediated by a quadratic term in the eddy energy budget.

The second derivative of this quadratic function is a self-adjoint linear operator that completely describes the structure of the energy exchange between mean and eddies.

We examine the structure of this operator, and the effect of discretization by finding eigenfunctions (Instantaneous Optimals/IOs) and eigenvalues (instantaneous energy growth rates).

We find eigenfunctions in the continuous problem by optimizing the energy growth rate over perturbations with unit energy. The Lagrangian is

$$I[u', v', b', \lambda] = B + S - \lambda(E - 1)$$

Energy: $\frac{1}{2} \int_{\Omega} (u')^2 + (v')^2 + (b')^2$

Energy Growth: $B = \int_{\Omega} v'b', \quad S = -\varepsilon \int_{\Omega} w'u'$

$$\nabla \cdot \boldsymbol{u}' + \varepsilon \partial_z w' = 0$$

The Euler-Lagrange equations are simple to derive:

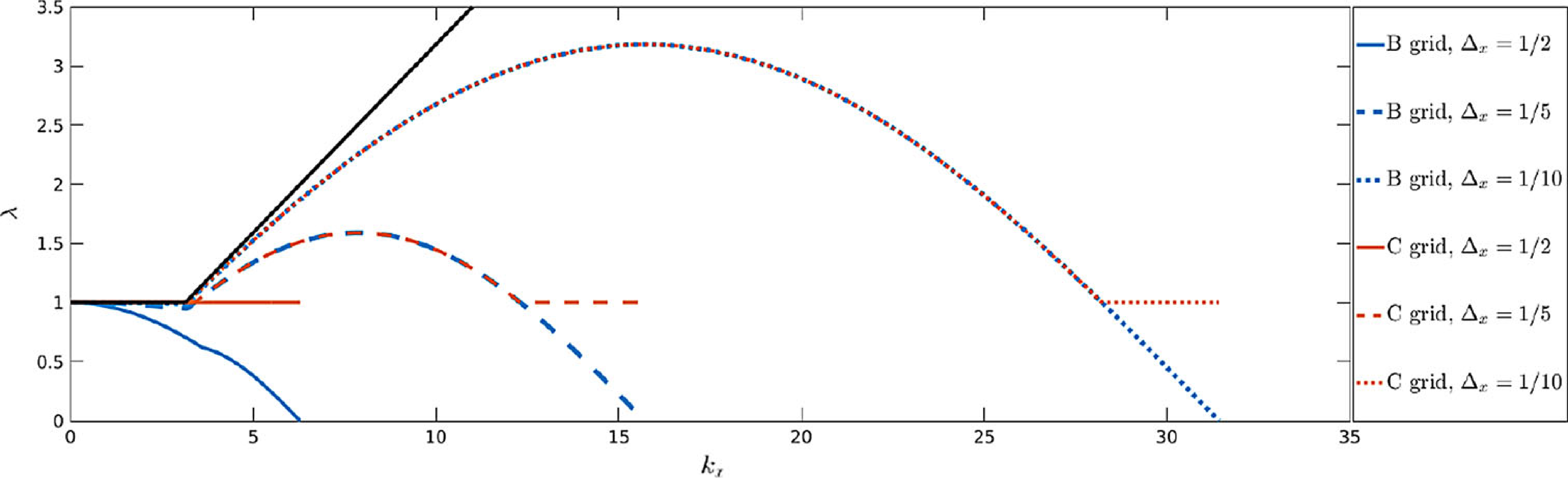
$$2\partial_x u' + \partial_y v' = \lambda \partial_z u'$$
$$\partial_z b' + \partial_y u' = \lambda \partial_z v'$$
$$v' = \lambda b'.$$

Easy to solve analytically.

Results are independent of Richardson number.

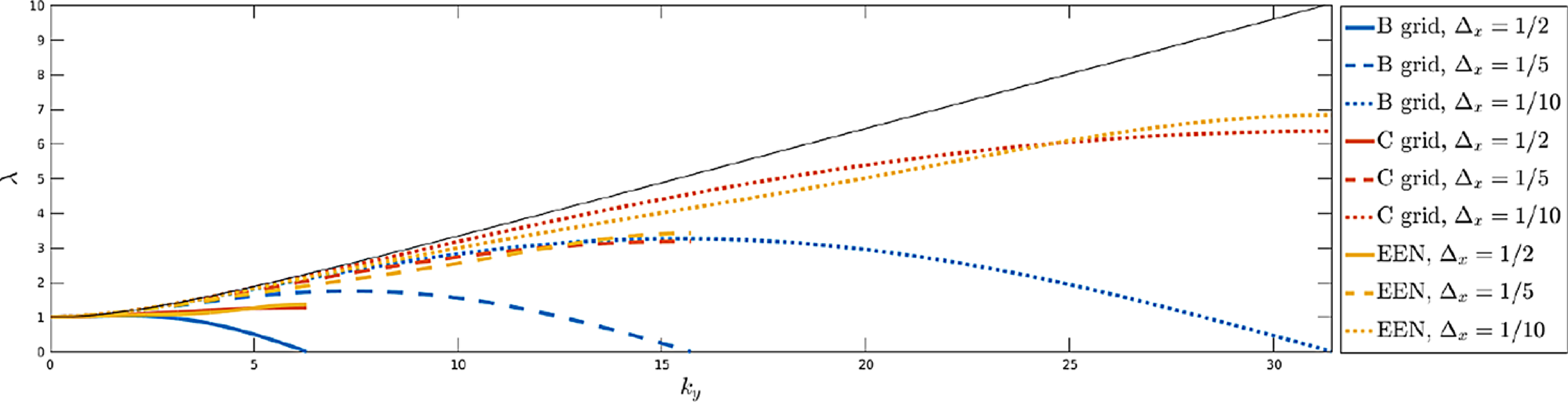
Large-scale growth dominated by baroclinic conversion, even on symmetric axis.

Small-scale growth dominated by shear production, even on baroclinic axis, and growth rates increase linearly.



Baroclinic Axis:

- Discrete models (color) have lower growth rates than exact (black)
- C grid (both versions identical) is slightly better at lower resolution, but overall quite similar



Symmetric Axis:

- Discrete models (color) have lower growth rates than exact (black)
- B Grid (blue) is worse than C Grid
- The two C Grid discretizations (red & yellow) are very similar

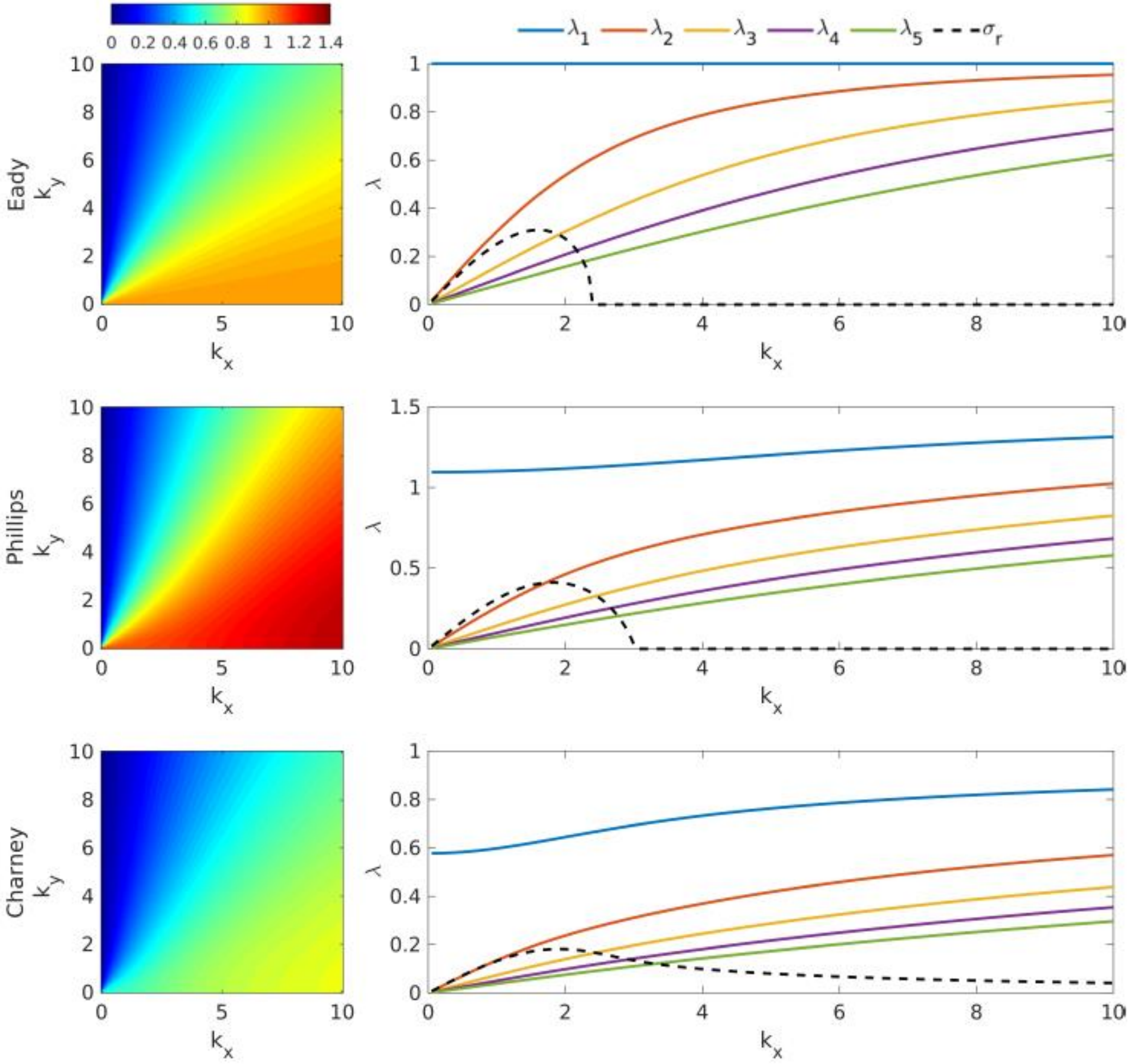
Thanks!

- Barham, Bachman, and Grooms, Ocean Modelling 2018: Linear BCI & Discretization
- Barham and Grooms, Theoretical & Computational Fluid Dynamics 2019: Optimals for Hydrostatic Eady

It’s hard to connect the results on optimals in the hydrostatic model to intuition about the dynamics.

We eliminate shear production by turning to the QG model.

Follow the same procedure: Optimize energy growth subject to fixed energy.



We still have strong growth at small scales, but at least it's not growing without bound.

Barham & Grooms, JFM 2020.