Multirate time integration for conservative two-way coupled atmosphere-ocean models

Hamburg COMMODORE Conference

Museum am Rothenbaum, Hamburg (D)

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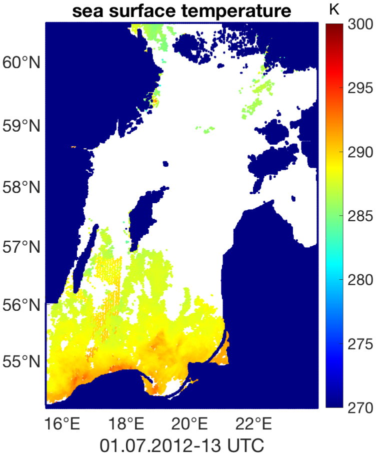
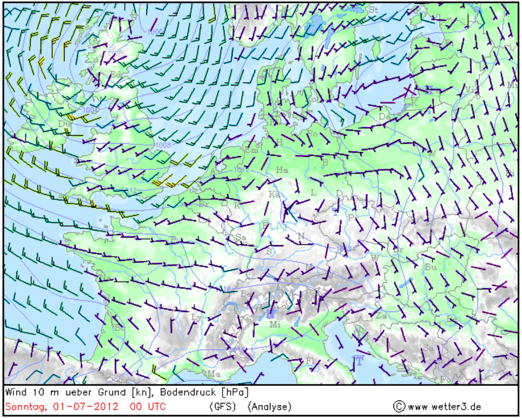
Coastal upwelling – Central Baltic Sea: July 01 - July 21, 2012



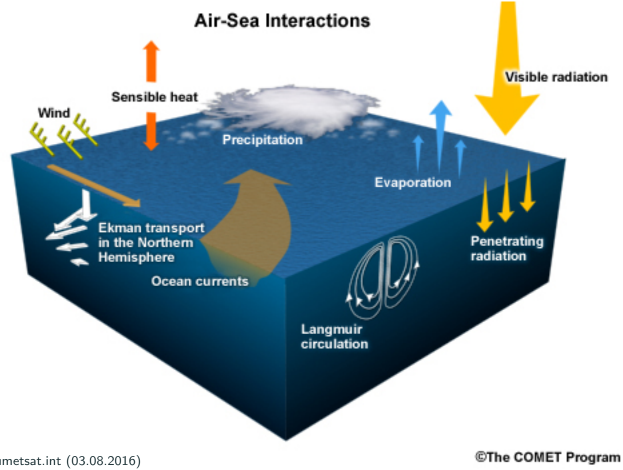
https://podaac.jpl.nasa.gov/ (21.02.2019)



Wind map of central Europe

www.wetter3.de (14.03.2019)

What happens at the water surface?



Linking of atmosphere and ocean via transfer of momentum, heat and gases e.g.

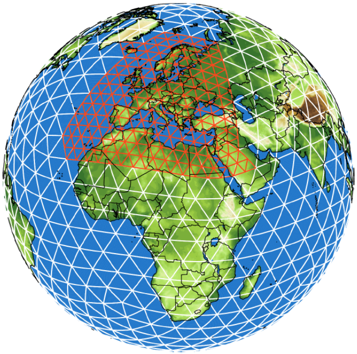
- Wind and atmospheric pressure generate waves and currents
- Ocean absorbs heat from the sun, greenhouse gases like carbon dioxide
- Warming/cooling of the atmosphere from below

www.eumetsat.int (03.08.2016)

ICON – ICOsahedral Non-hydrostatic modeling framework (Atmosphere)

- Developed by German Weather Service (DWD) and Max Planck Institute for Meteorology (MPI-M)
- Unified modeling system for global numerical weather prediction (NWP) and climate modeling
- Flexible grid nesting capability and usage of non-hydrostatic equations
- Operational weather forecast at DWD (13 km global + 6.5 km local resolution)
- Central Baltic Sea: approx. 2500 m

Zängl et al., 2015; Giorgetta et al., 2018



Icosahedral triangular horizontal grid with fairly uniform resolution on sphere and simple regional grid refinement

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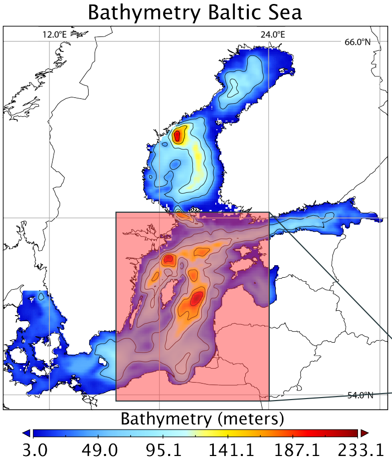
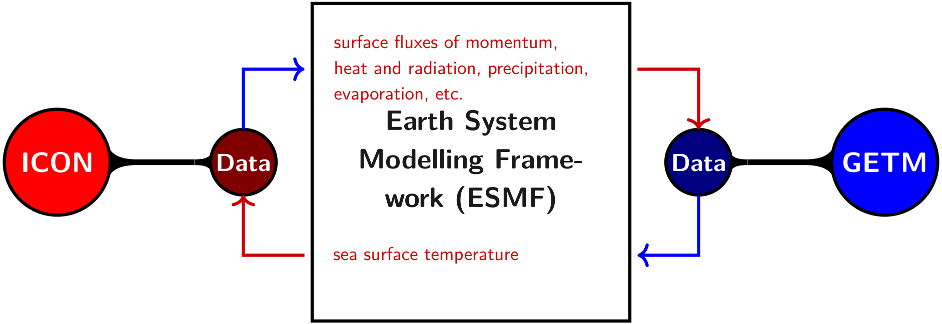
GETM – General Estuarine Transport Model (Baltic Sea)



- Co-developed at Leibniz Institute for Baltic Sea Research (IOW)
- Modeling baroclinic bathymetry-guided flows including drying and flooding processes
- Reproducing baroclinic features such as upwelling, internal seiches and stratified flows
- Simulating flows and transport on larger scales than estuarine scales, e.g. salt water inflows in the Baltic Sea
- Usage of structured rectangular grid
- *Area of interest: Central Baltic Sea* (approx. 600 m)

Burchard et al., 2004; Holtermann et al., 2014; Klingbeil et al., 2018

How are ICON and GETM online coupled?



- Which variables will be exchanged?
- Which time intervals will be suitable for a data exchange?
- Which interpolation method will best fit for a data exchange?

Realisation of air-sea interactions in ICON & GETM

ICON:

Momentum:

$$\tau_s^x = -\rho \cdot C_m^d \cdot |\mathbf{v}| \cdot u$$
$$\tau_s^y = -\rho \cdot C_m^d \cdot |\mathbf{v}| \cdot v$$

Heat:

$$Q = Q_s + Q_l + Q_b + Q_{SW}$$

GETM:

Momentum:

$$\tau_s^x = \rho \cdot C_m^d \cdot |\mathbf{v}| \cdot u$$
$$\tau_s^y = \rho \cdot C_m^d \cdot |\mathbf{v}| \cdot v$$

Heat:

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Heat:

$$Q = Q_s + Q_l + Q_b + Q_{SW}$$

No mass exchange with ocean via precipitation and evaporation due to exact local mass conservation.

Ullrich et al., 2017

GETM:

Momentum:

$$\tau_s^x = \rho \cdot C_m^d \cdot |\mathbf{v}| \cdot u$$
$$\tau_s^y = \rho \cdot C_m^d \cdot |\mathbf{v}| \cdot v$$

Heat:

$$Q = Q_s + Q_l + Q_b$$

Considering of precipitation and evaporation for fresh water flux.

Development of different atmosphere-ocean models

- Models for studies of air-sea interactions
  1D: Studying mass, momentum and energy coupling between atmosphere and ocean with a water/air column model system
  2D: Constructing an idealised coupled model system with straight coast and upwelling favourable winds
  3D: Fully coupled atmosphere-ocean experiment (Central Baltic Sea with ICONGETM) Bauer et al. in prep.
- Utilising different strategies for asynchronous or synchronous coupling
  a) Derivation and application of numerical methods with multirate approaches for idealised atmosphere-ocean models
  b) Online coupling with ESMF for ICONGETM

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Why joined equations for atmosphere-ocean systems?: Continuity equation

Atmosphere:

- Dry air (d):
  $\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \cdot \mathbf{v}_d) = 0$
- All other components k:
  $\frac{\partial \rho_k}{\partial t} + \nabla \cdot (\rho_k \cdot \mathbf{v}_k) = \sigma_k$

$$\boxed{\frac{\partial \rho^A}{\partial t} + \nabla \cdot (\rho^A \cdot \mathbf{v}^A) = \sum [\sigma_k] = S}$$

Ocean:

- Fresh water (f):
  $\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \cdot \mathbf{v}_f) = \sigma_f$
- Salinity (sa):
  $\frac{\partial \rho_{sa}}{\partial t} + \nabla \cdot (\rho_{sa} \cdot \mathbf{v}_{sa}) = 0$

$$\boxed{\frac{\partial \rho^O}{\partial t} + \nabla \cdot (\rho^O \cdot \mathbf{v}^O) = \sigma_f}$$

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$$\boxed{\frac{\partial \rho^O}{\partial t} + \nabla \cdot (\rho^O \cdot \mathbf{v}^O) = \sigma_f}$$

Mass conserving: $\frac{\partial (\rho^A + \rho^O)}{\partial t} + \nabla \cdot (\rho^A \cdot \mathbf{v}^A + \rho^O \cdot \mathbf{v}^O) = S + \sigma_f = 0$

Mass conservation of atmosphere-ocean system:
$\Rightarrow$ atmosphere and ocean, each on its own not mass conserving
$\Rightarrow$ compressible and non-hydrostatic set of equation

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General structure of governing equations

1. No difference in generalized equation for atmosphere and ocean:

$$\frac{\partial(\rho\psi)}{\partial t} = -\nabla\cdot\left(\rho\psi\cdot\mathbf{v}^T\right) + \sum_{k\in\mathcal{M}}\left[-\nabla\cdot\left(\psi_k\cdot\mathbf{J}_k^T+\lambda_k\right)+\sigma_k\right] \tag{1}$$

2. $\mathcal{M}$ substances (mass density (ρ) respectively for atmosphere and ocean)

   Atmosphere: e.g. dry air, water vapour, rain drops, etc.

   Ocean: fresh water and salinity

3. ψ being a free-variable parameter for

   Atmosphere: mass $\psi := 1$, flow velocity $\psi := \mathbf{v}^A$ and total energy $\psi := e^A$

   Ocean: mass $\psi := 1$, flow velocity $\psi := \mathbf{v}^O$ and total energy $\psi := e^O$

4. Diffusive mass flux ($\mathbf{J}$), flux (λ) and external source (σ) for ψ_k

General structure of governing equations

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2. General structure of differential equation where equation (1) holds for both, atmosphere and ocean

$$\frac{\partial y_O}{\partial t} = f(y) \qquad \text{and} \qquad \frac{\partial y_A}{\partial t} = g(y)$$

$$\frac{\partial y}{\partial t} = \frac{\partial y_O}{\partial t} + \frac{\partial y_A}{\partial t} = f(y) + g(y) = F(y)$$

Why joined equations for atmosphere-ocean systems?

Advantages:

- Synchronous online coupling
- Direct physical representation of air-sea interactions, i.e. no parameterization at interface

Challenges:

- Supporting different time steps for integration
- Different horizontal resolutions

for both parts, atmosphere and ocean

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$\Rightarrow$ *Multirate time integration allow splitting into two and more parts for joined atmosphere-ocean systems with own characteristic numerical representation.*

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Variety of multirate methods in atmosphere and ocean models

Examples of already applied multirate approaches/methods:

- Split-explicit methods for separating stiff and non-stiff components
- Predictor-Corrector method for splitting of dynamical core and tracer advection, fast-physics parametrizations and horizontal diffusion (ICON) Zängl et al., 2015
- Splitting in barotropic-baroclinic modes (e.g. GETM) Klingbeil et al., 2018
- ...

Explicit Runge–Kutta method (eRK)

eRK method for solving of equation $\frac{\partial y}{\partial t} = f(y) + g(y) = F(y)$:

$$
\begin{aligned}
Y_i &= y_n + h \sum_{j=1}^{i-1} [a_{ij} F(Y_j)] \\
y_{n+1} &= y_n + h \sum_{i=1}^{s} [b_i F(Y_i)]
\end{aligned}
$$

or

$$
\begin{aligned}
Z_i(0) &= y_n \\
\frac{\mathrm{d}Z_i(\tau)}{\mathrm{d}\tau} &= \sum_{j=1}^{i-1} [a_{ij} F(Y_j)] \\
Y_i &= Z_i(h) \\
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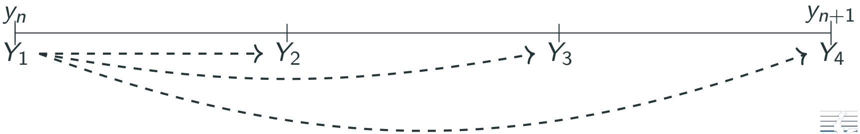
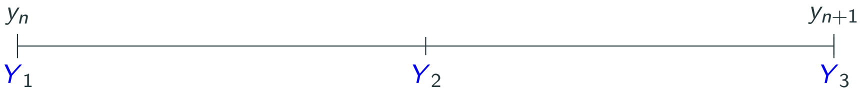


Multirate infinitesimal step method (MIS)

General idea of MIS for equation (2), i.e. $\dot{y} = f(y) + g(y)$:

Wensch et al., 2009, Knoth et al., 2014, Bauer et al., 2019



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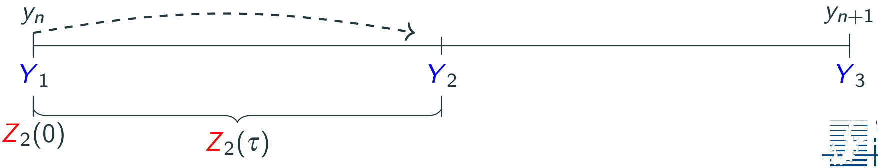

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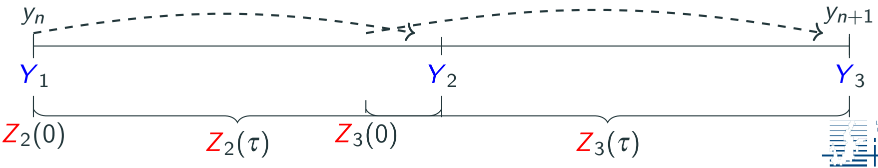

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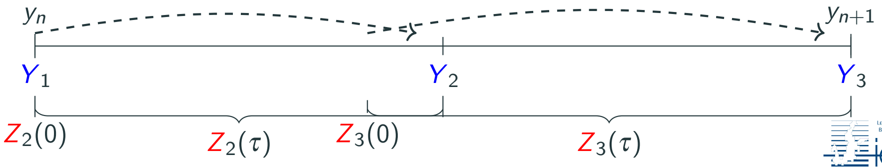
$$
\begin{aligned}
Z_i(0) &= y_n + \sum_{j=1}^{i-1} \left[\alpha_{ij}\left(Y_j - y_n\right)\right] \\
\frac{\mathrm{d}Z_i(\tau)}{\mathrm{d}\tau} &= \frac{1}{h}\sum_{j=1}^{i-1} \left[\gamma_{ij}\left(Y_j - y_n\right)\right] + \sum_{j=1}^{i-1} \left[\beta_{ij} f\left(Y_j\right)\right] + d_i g\left(Z_i(\tau)\right) \\
Y_i &= Z_i(h) \qquad i = 1, \ldots, s+1 \\
y_{n+1} &= Y_{s+1}
\end{aligned}
$$

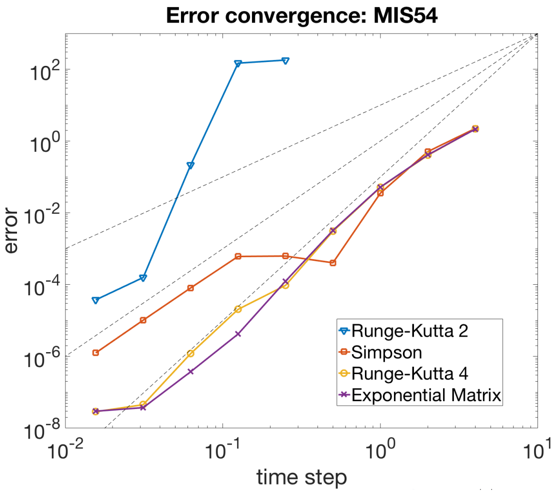


Multirate infinitesimal step method (MIS)

- Further splitting of fast part (e.g. atmosphere) possible, i.e.

$$\dot{y} = f(y) + \sum_{k_A=1}^{N_A} g_{k_A}(y)$$

- Example method MIS54: 4th-order Bauer et al., 2019
- Applied to purely atmospheric examples, e.g. Cold Bubble Test
- 1D AO simulations with more efficient time step, i.e. $1\,s$ and $2\,s$ vs. $10\,s$
- 2D AO simulations ...
  missing appropriate test cases / benchmark examples for atmosphere-ocean models



Conclusions

- Requirement of correct physical representation of air-sea interaction for high resolved coupled atmosphere-ocean models, e.g. for ICONGETM
- Studying of air-sea interaction with 1D and 2D scenarios suggest joined atmosphere-ocean simulations
- MIS method overcomes challanges for joined atmosphere-ocean models

Conclusions

- Requirement of correct physical representation of air-sea interaction for high resolved coupled atmosphere-ocean models, e.g. for ICONGETM
- Studying of air-sea interaction with 1D and 2D scenarios suggest joined atmosphere-ocean simulations
- MIS method overcomes challanges for joined atmosphere-ocean models

But: *Purely explicit integration of slow part, maybe the ocean,*
with the longest time step due to underlying explicit Runge–Kutta method in MIS

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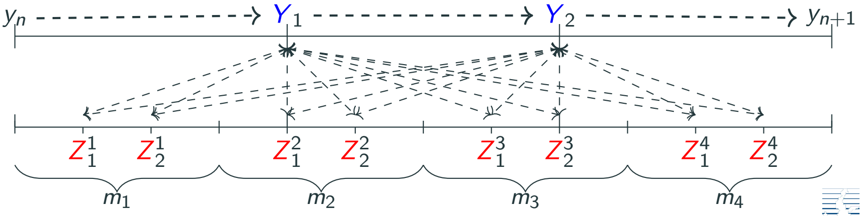
Multirate generalized additive Runge–Kutta method (MGARK)

MGARK method for solving of equation (2), i.e. $\dot{y} = \sum_{k_O=1}^{N_O} f_{k_O}(y) + \sum_{k_A=1}^{N_A} g_{k_A}(y)$: Günther et al., 2016

Structure for only two parts:

$$y_{n+1} = y_n + h \sum_{i=1}^{s_f} \left[b_i^f f\left(Y^i\right)\right] + h \sum_{\lambda=1}^{N} \sum_{i=1}^{s_g} \left[m_\lambda b_i^g g\left(Z_i^\lambda\right)\right]$$



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Y_i &= y_n + h \sum_{j=1}^{s_f} \left[a_{ij}^{f,f} f\left(Y_j\right) \right] + h \sum_{\lambda=1}^{N} \sum_{j=1}^{s_g} \left[m_\lambda a_{ij}^{f,g,\lambda} g\left(Z_j^\lambda\right) \right], \qquad i = 1, \ldots, s_f \\
Z_i^\lambda &= y_n + h \sum_{j=1}^{s_f} \left[a_{ij}^{g,f,\lambda} f\left(Y_j\right) \right] + h \sum_{l=1}^{\lambda-1} \sum_{j=1}^{s_g} \left[m_l b_j^g g\left(Z_j^l\right) \right] + \\
&\quad h \sum_{j=1}^{s_g} \left[m_\lambda a_{ij}^{g,g} g\left(Z_j^\lambda\right) \right], \qquad i = 1, \ldots, s_g, \qquad \lambda = 1, \ldots, N \\
y_{n+1} &= y_n + h \sum_{i=1}^{s_f} \left[b_i^f f\left(Y_i\right) \right] + h \sum_{\lambda=1}^{N} \sum_{i=1}^{s_g} \left[m_\lambda b_i^g g\left(Z_i^\lambda\right) \right]
\end{aligned}
$$

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